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Equality of Opportunity Research Series #101
November 2026

University of Zurich[UZH]

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URPP Equality of Opportunity Discussion Paper Series No. 101, November 2026

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Income Anonymity*

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November 17, 2026

Abstract

Policies are routinely evaluated based on criteria that are symmetric in individual incomes. We show that, when people have heterogeneous preferences over prices, there exists at most one Paretian ranking that is consistent with this symmetry property. We show that such a criterion exists in many canonical examples, but not in general. When the axioms are in conflict, our characterization gives rise to a family of social welfare criteria whose members correspond to different ways of relaxing the symmetry requirement. Under some assumptions, these criteria imply rankings of income distributions that can be estimated based on consumer expenditure data. We estimate that, given current expenditure patterns in the United States, these criteria rank income distributions approximately according to the sum of log incomes.

JEL Classification: D63, D31, H24
Keywords: money-metric utility, additive separability, inequality aversion, anonymity, income redistribution, social welfare function

*We are grateful for helpful conversations with Benoit Decerf, Matthias Doepke, Simon Fuchs, Antoine Germain, Damian Kozbur, Aart Kraay, François Maniquet, Erwin Ooghe, Berk Özler, Nicola Persico, Paolo Piacquadio, Uzi Segal, Itai Sher, Peter Wakker and John Weymark, and for comments from seminar participants at the University of Oklahoma, the University of Oslo, the Atlanta Fed, the University of Cologne, the Welfare Economics and Economic Policy virtual seminar, Columbia University, Birkbeck University of London, the University of Zurich, Pennsylvania State, the PSE workshop on "Around and Beyond GDP", Virginia Commonwealth University, Yale, the University of Pennsylvania and the University of Arizona. We gratefully acknowledge financial support from the University of Zurich Research Priority Program (URPP) "Equality of Opportunity."

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1 Introduction

Policies are routinely evaluated by criteria that treat people with the same income symmetrically. For example, the U.S. Congressional Budget Office weights all monetary gains equally; the U.K. Treasury uses distributionally-weighted assessments that place greater value on income gains at the bottom of the income distribution; and the World Bank measures poverty with a common monetary threshold.[1] These criteria differ in terms of how they treat gains and losses at different income levels. However, common to all of them is the symmetric treatment of individuals conditional on income: if Ann and Barbara have the same income (and share other relevant demographic characteristics, such as family status), then increasing Ann's income by a dollar is considered just as good as increasing Barbara's income by a dollar — even if Ann and Barbara have different tastes that lead them to spend their incomes in different ways.

In this paper, we study the broader implications of this symmetry property. Our key axiom is *Income Anonymity*, which requires the social ordering to be indifferent with respect to permutations of the income distribution, regardless of prices. We supplement this axiom with the standard Pareto condition. In our context, Pareto means that a simultaneous change in incomes and in prices that benefits everyone should be considered socially desirable. We show that, in the presence of preference heterogeneity, there exists *at most one* ranking that satisfies both conditions. In particular, while there are many evaluation criteria that are symmetric and increasing in incomes, there is at most one such criterion that is consistent with Pareto. Surprisingly, the Pareto condition pins down a unique way for trading off gains and losses at different parts of the income distribution.

We demonstrate the implications of this result with four canonical examples. In these examples, our theorem provides concrete guidelines on how to rank income distributions conditional on prices. The social welfare function depends on what we assume about the profile of individual preferences:

- **Homothetic Preferences:** Income distributions are ranked according to their geometric mean.
- **Stone-Geary Preferences:** Income distributions are ranked according to the geometric mean of the surpluses over subsistence levels.
- **Gorman Form Preferences:** Income distributions are ranked according to their arithmetic mean.

[1] See the U.S. Office of Management and Budget's cost-benefit analysis framework [Office of Management and Budget, 2023], the U.K. Treasury's use of distributional weights [HM Treasury, 2022, Adler, 2016]. See also the World Bank's focus on poverty reduction using real-income thresholds [World Bank, 2020].

- **Price Independent Generalized Linearity (PIGL) Preferences:** Income distributions are ranked according to an Atkinson social welfare function, with an inequality aversion parameter equal to one minus the Engel elasticity parameter.

There are also preference profiles for which Income Anonymity is inconsistent with Pareto. In such cases, our characterization corresponds to a family of social orderings that satisfy weaker versions of our axioms. In the final part of the paper, we use the United States Consumer Expenditure Survey to calibrate these social welfare criteria. We find that they are closely aligned, and that they all rank income distributions roughly according to the sum of log incomes.

This paper contributes to a rich literature on the axiomatic characterization of social preferences. The approach here is closely related to the money metric utility approach [Deaton and Muellbauer, 1980, Fleurbaey, 2009, Fleurbaey and Maniquet, 2011, 2018]. The money metric utility approach proposes a social preference relation that is an aggregation of individuals' equivalent incomes at common reference prices ("money metric utilities"). There are many social welfare functions that satisfy anonymity with respect to the distribution of money metric utilities. These criteria trivially satisfy Income Anonymity conditional on the reference prices. The anonymity conditions that we consider here are more restrictive, as they require anonymity with respect to the distribution of income even when prices deviate from their reference levels.

This paper is also related to the literature on price-independent welfare prescriptions [Roberts, 1980, Slesnick, 1991, Blackorby et al., 1993, Fleurbaey and Blanchet, 2013]. Roberts [1980] studies the conditions under which income distributions can be ranked irrespective of prices, and finds that they are "highly restrictive." This sparked a debate about whether it is appropriate to require welfare prescriptions to be independent of prevailing prices [Fleurbaey and Blanchet, 2013]. The social preference relation that we derive generates price-independent welfare prescriptions only in special cases. Generally, the way it ranks income distributions will depend on the price levels.

In addition, this paper is related to the literature on the inconsistency of Pareto and other normative principles. Sen [1970a] and Kaplow and Shavell [2001] show that the Pareto principle leaves limited room for expressing a concern for non-welfarist normative principles such as liberalism, justice, or procedural fairness (see also Sher [2024]). In the context of resource allocation problems, Sen [1970b], Suzumura [1981a], Suzumura [1981b], Suzumura [1983], Tadenuma [2002] and Fleurbaey and Trannoy [2003] uncover tensions between Pareto and various egalitarian principles pertaining to the fair allocation of resources. We add to these impossibility results by showing that, for some preference profiles, Pareto is also inconsistent with Income Anonymity. However, in light of the negative results in this literature,

our positive results are perhaps more surprising: we show that, in some important special cases, there is no conflict between Income Anonymity and Pareto, even when preferences are heterogeneous.

Finally, this paper is related to the literature on the axiomatic foundations of additively-separable utility functions [Gorman, 1968], [Wakker, 1989], [Blackorby et al., 1998], [Qin and Rommeswinkel, 2024]. Here, we establish that if the social preference relation satisfies Income Anonymity and Pareto, then it must be additively separable in individual budgets. Previous results in this literature obtain additively-separable representations by assuming that it is possible to rank subgroups of variables independently from one another. In our analysis, this is embedded in the Pareto condition: given that, in our framework, all goods are private goods, Pareto implies that each individual's consumption bundles can be ranked independently of other individuals' consumption bundles.[2]

2 Environment

There are $2 \leq I < \infty$ individuals and $2 \leq J < \infty$ goods. Throughout, we use subscripts for indicating individuals and superscripts for indicating goods. A budget is a combination of an income, $m \in \mathbb{R}_{++}$, and a price vector, $p = (p^1, ..., p^J) \in \mathcal{P} \subseteq \mathbb{R}^J_{++}$. Here, $\mathcal{P}$ is the domain of prices over which both individual and social preferences are formally defined. We assume that $\mathcal{P}$ is nonempty and open.

2.1 Individual preferences

Consumption bundles are denoted $x = (x^1, ..., x^J) \in \mathbb{R}^J_+$. Individuals' preferences over consumption bundles are continuous, strictly increasing (that is, if $x^j > y^j$ for every j, then $x \succ y$), and strictly convex, and are represented by continuous utility functions, $\{u_i\}_{i=1}^I$. For each $1 \leq i \leq I$, let $v_i : \mathbb{R}_{++} \times \mathcal{P} \to \mathbb{R}$ denote the indirect utility function,

$$v_i(m, p) = \max_x u_i(x) \text{ s.t. } \sum_{j=1}^{J} p^j x^j \leq m \tag{1}$$

We use $\preceq_i$ (with corresponding strict relations, $\prec_i$ and $\succ_i$, and indifference relation, $\sim_i$) to denote the preference relation represented by v_i (where individual preferences are defined over budgets in $\mathbb{R}_{++} \times \mathcal{P}$). Note that, given the assumption that preferences over consumption bundles are strictly increasing, each v_i is strictly increasing in m.

[2] This way of obtaining additive separability is similar to Gustafsson et al. [2023].

For our purposes, it is convenient to focus on the *money-metric utility* (or "equivalent income") representations of individual preferences, at some reference prices, $p_0 \in \mathcal{P}$. For our application, we specify the reference price vector as the vector of equilibrium prices. However, our theoretical results do not depend on this choice: one is free to specify the reference prices in any way they see fit.[3]

To define the money-metric utility representations, let $e_i(m, p_S, p_T)$ denote the solution to the indifference condition,

$$(m, p_S) \sim_i (e_i(m, p_S, p_T), p_T)$$

Here, p_S are the "source" prices, and p_T are the "target" prices. The quantity $e_i(m, p_S, p_T)$ is individual i's equivalent income at the target prices, given a "source" budget of (m, p_S).[4]

Note that $e_i(\cdot, \cdot, p_0)$ is a representation of $\preceq_i$. Without loss of generality, we assume that

$$v_i(m, p) = e_i(m, p, p_0)$$

that is, that the indirect utility functions are already in their money-metric utility form. In addition, we assume that the functions $\{e_i\}_{i=1}^I$ are continuously differentiable.

Let $x_i(m, p) = (x_i^1(m, p), ..., x_i^J(m, p))$ denote individual i's optimal consumption bundle given the budget (m, p).[5] To simplify notation, we use the shorthand $x_i^j(m) := x_i^j(m, p_0)$.

Define the function $\Delta x_{i,i'}^j(\cdot)$ as

$$\Delta x_{i,i'}^j(m) := x_i^j(m) - x_{i'}^j(m)$$

The function $\Delta x_{i,i'}^j(\cdot)$ measures the difference between individual i's Engel curve and individual i''s Engel curve with respect to good j, given the reference prices, p_0.

Throughout, we make the following assumption.

Assumption 1. *For every good j and individuals $i \neq i'$, it holds that either $\Delta x_{i,i'}^j(\cdot) > 0$ or $\Delta x_{i,i'}^j(\cdot) < 0$.*

This assumption means that, conditional on the reference prices, no two individuals with the same income consume exactly the same quantity of any good. In Appendix C.1, we

[3] Fleurbaey and Maniquet, 2018, discuss how different ethical views can be expressed by different choices of the reference prices.

[4] The existence and uniqueness of $e_i(m, p_S, p_T)$ follow from our assumptions on individual preferences. Given continuity, existence follows from the intermediate value theorem: for a low enough m_T, the budget (m, p_S) is preferred over the budget (m_T, p_T), and for a high enough m_T, the budget (m_T, p_T) is preferred over the budget (m, p_S). Thus, there exists some intermediate $m_T = e_i(m, p_S, p_T)$ such that the individual is indifferent. Uniqueness follows from the strict monotonicity of preferences.

[5] Note that, given the assumption that preferences are strictly convex and u_i is continuous, the consumption bundle $x_i(m, p)$ is uniquely defined.

discuss the role of this assumption and how it can be relaxed.

2.2 Evaluation guidelines

A decision maker is tasked with designing policy evaluation guidelines, which will be used for making policy decisions as they arise. For example, we can think of the decision maker's task as designing guidelines for the Congressional Budget Office on how to decide whether or not to recommend certain policy proposals, depending on how they are going to change the income distribution and how they are going to affect prices. Alternatively, we can think of the decision maker as an advisor to the World Bank, that needs to specify guidelines for evaluating and prioritizing different development projects, depending on how they will affect prices and incomes.

Formally, let Ω denote the set of all rankings of elements of the form $(\mathbf{m}, p)$, where $\mathbf{m} = (m_1, ..., m_I) \in \mathbb{R}^I_{++}$ is the distribution of income and $p \in \mathcal{P}$ is a vector of (common) prices. In addition, let D denote the domain of profiles of individual preferences, that is, vectors of individual rankings of the form $(\preceq_1, ..., \preceq_I)$, where $\preceq_i$ is individual i's ranking over budgets. Here, we interpret D as the set of empirically-relevant profiles that the decision maker ought to consider when specifying the evaluation guidelines. We assume that if a preference profile is in D, then any permutation of that preference profile is also in D:

$$(\preceq_1, ..., \preceq_I) \in D \Rightarrow (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)}) \in D \text{ for any permutation } \sigma$$

Evaluation guidelines are a function, $\preceq^0$: $D \to \Omega$, that specifies the social ranking of income distributions and prices as a function of individual preferences. Here, the social ranking $\preceq^0 (\preceq_1, ..., \preceq_I)$ constitutes guidelines on how to rank income distributions and prices when the profile of individual preferences is $(\preceq_1, ..., \preceq_I)$.[6] The statement

$$(\mathbf{m}, p) \preceq^0 (\preceq_1, ..., \preceq_I)(\mathbf{m}', p')$$

means that, conditional on the preference profile $(\preceq_1, ..., \preceq_I)$, policy makers ought to consider $(\mathbf{m}', p')$ to be a weakly more desirable outcome than $(\mathbf{m}, p)$.

[6]Note that we are restricting attention to policy guidelines that correspond to conditional rankings. The key assumption is that if A is preferred over B conditional on some preference profile, and B is preferred over C conditional on the same preference profile, then A ought to be preferred over C conditional on that profile.

2.2.1 Axioms

Our main results concern the ranking of budget allocations conditional on a fixed preference profile. This analysis builds on two axioms: (single-profile) Pareto and *Income Anonymity*, which requires the evaluation criterion to be symmetric in individual incomes. These axioms will be formally introduced in Section 2.3. However, to motivate these axioms and relate them to the broader question of how to design policy evaluation guidelines that may be contingent on individuals' preferences, we begin by presenting several axioms in the multi-profile setting.

The first axiom is the Classical Anonymity condition. To state this condition, it is convenient to introduce the notation

$$\sigma(\mathbf{m}) = (m_{\sigma(1)}, ..., m_{\sigma(I)})$$

where $\sigma : \{1, ..., I\} \to \{1, ..., I\}$ is a permutation.

Classical Anonymity is stated as follows.

Axiom (Classical Anonymity)**.** *For any permutation σ and profile $(\preceq_1, ..., \preceq_I) \in D$,*

$$(\mathbf{m}, p) \preceq^0 (\preceq_1, ..., \preceq_I)(\mathbf{m}', p) \Leftrightarrow (\sigma(\mathbf{m}), p) \preceq^0 (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})(\sigma(\mathbf{m}'), p)$$

This axiom means that we are restricting attention to subpopulations that share similar observable characteristics that may be relevant for the decision maker. For example, we might ask how to allocate childcare vouchers among non-veteran, able-bodied working single mothers. We are *not* asking to what extent we should prioritize single mothers over married mothers, veterans over non-veterans, or disabled people over able-bodied people. Rather, we are interested in ranking budget allocations conditional on holding other relevant observable characteristics fixed. Classical Anonymity defines such an environment by imposing that there are no relevant observable characteristics which are encoded in the index number (eg, "person 1 is a disabled veteran and person 2 is not").

The second axiom is an information restriction, which limits the extent to which the evaluation guidelines can be made contingent on individuals' preferences.

Axiom (Independence of Irrelevant Prices)**.** *For any permutation $\sigma : \{1, ..., I\} \to \{1, ..., I\}$ and two profiles $(\preceq_1, ..., \preceq_I), (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)}) \in D$,*

$$(\mathbf{m}, p) \preceq^0 (\preceq_1, ..., \preceq_I)(\mathbf{m}', p) \Leftrightarrow (\mathbf{m}, p) \preceq^0 (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})(\mathbf{m}', p)$$

This condition is a weaker version of Arrow's Independence of Irrelevant Alternatives

axiom Arrow, 1950.[7] It says that, in order to choose between two income distributions, it is not necessary to know the exact profile of individual preferences over budgets that are not being considered. Instead, aggregate information about the statistical distribution of these preferences should suffice.

This informational restriction arises naturally in environments in which individuals' preferences are their own private information. To demonstrate, consider a simple example in which a public official must allocate a daycare voucher to one of two non-veteran, able-bodied working single mothers, Ann or Barbara. The public official has access to their tax returns, which allows him to base his decision on their respective income levels. Given that both women have children that are already enrolled in full-time childcare, receiving the daycare voucher would amount to a one-to-one increase in their disposable income. Thus, the question is whether to increase Ann's income or Barbara's income by the same amount.

Now assume that — in violation of the above axiom — the guidelines require the public official to give the benefit to Ann *if and only if* she prefers asparagus over beets. Putting aside the oddity of these guidelines, there is a question of whether it would even be possible for the public official to follow them. Knowing the guidelines, Ann has a clear incentive to report a preference for asparagus, regardless of the truth. Her self-reported preferences therefore contain no valuable information, and it may be impossible to ascertain her true preferences.

Independence of Irrelevant Prices rules out such guidelines. This reflects the reality that it may not be possible to rank income distributions in a way that is contingent on the exact preference profile, because, in such situations, individuals have no incentive to truthfully reveal their preferences. We develop this point formally in Appendix A.

Note that the plausibility of this informational restriction depends on the form of preference heterogeneity that we consider. One might object to this axiom on the grounds that *some* preferences may be both normatively relevant and observable. For example, we can probably learn about Ann and Barbara's preferences for cheaper medical care based on their medical records. As these preferences may be informative of their underlying health conditions, this information might compel us to prioritize one of them over the other. *If* the price of medical care varies across the price vectors in $\mathcal{P}$, this would be inconsistent with our axiom. However, if $\mathcal{P}$ contains only price vectors that vary in the relative prices of

[7] Arrow's axiom says that the only relevant information for ranking $(\mathbf{m}, p)$ and $(\mathbf{m}', p)$ is a list of which individuals prefer $(\mathbf{m}, p)$ to $(\mathbf{m}', p)$, which individuals prefer $(\mathbf{m}', p)$ to $(\mathbf{m}, p)$, and which individuals are indifferent between the two options. Given our assumptions, this information is the same for any profile in D, as individual i prefers $(\mathbf{m}, p)$ to $(\mathbf{m}', p)$ if and only if $m_i \geq m_i'$. Thus, Arrow's axiom demands the ranking of $(\mathbf{m}, p)$ and $(\mathbf{m}', p)$ to be independent of the preference profile. Here, we consider a weaker requirement, which allows the ranking to differ among two preference profiles that do not amount to a permutation of one another.

vegetables — holding the cost of medical care constant[8] — then information about medical expenditures is *not* information about preferences over budgets in $\mathbb{R}_{++} \times \mathcal{P}$. Rather, this information is treated just like any other observable characteristic (such as single motherhood), and the policymaker can condition on it directly. Substantively, Independence of Irrelevant Prices applies whenever there is at least some preference information that is unobservable and unverifiable.

Finally, the decision maker would like to specify guidelines that satisfy the standard (multi-profile) Pareto condition.

Axiom (Multi-Profile Pareto)**.** *1. If* $(m_i, p) \preceq_i (m_i', p')$ *for every* i*, then* $(\mathbf{m}, p) \preceq^0 (\preceq_1, ..., \preceq_I)(\mathbf{m}', p')$.

2. If, in addition, $(m_i, p) \prec_i (m_i', p')$ *for some* i*, then the social preference above is strict.*

This condition is the standard unanimity condition. It requires that if everyone prefers $(\mathbf{m}', p')$ over $(\mathbf{m}, p)$, then the social ordering should reflect this consensus.

2.3 Single-profile implications

We now clarify what our axioms imply for the evaluation guidelines $\preceq^0 (\preceq_1, ..., \preceq_I)$, that is, the ranking of budget allocations in $\mathbb{R}^I_{++} \times \mathcal{P}$ conditional on the preference profile $(\preceq_1, ..., \preceq_I)$. To simplify notation, we fix some $(\preceq_1, ..., \preceq_I)$ and denote this ranking simply by $\preceq$ (with no subscript, to differentiate from the individuals' preference relations). Note that $\preceq$ is a social ranking of budget allocations in $\mathbb{R}^I_{++} \times \mathcal{P}$.

Consider the following two properties, concerning a ranking $\preceq \in \Omega$ (with the corresponding indifference relation $\sim$ and the strict preference relation $\prec$). The first is Income Anonymity, which states that income distributions can be ranked without knowing which income goes to which individual.

Axiom (Income Anonymity)**.** *For every* $p \in \mathcal{P}$*,* $\mathbf{m} \in \mathbb{R}^I_{++}$*, and permutation* σ*, it holds that*

$$(\mathbf{m}, p) \sim (\sigma(\mathbf{m}), p)$$

This axiom states that the ranking of income distributions cannot depend on which income belongs to which individual: for example, a situation in which Ann has 50 dollars and Barbara has 100 dollars should be considered no better and no worse than a situation in which Barbara has 50 and Ann has 100.

The following claim establishes that this property is implied by the combination of Independence of Irrelevant Prices and Classical Anonymity:

[8] Formally, this amounts to assuming that medical care is not among the goods $1, ..., J$.

Claim 1. *Assume that the evaluation guidelines, $\preceq^0$, satisfy Independence of Irrelevant Prices and Classical Anonymity. Then, for every profile $(\preceq_1, ..., \preceq_I)$, the ranking $\preceq^0$ $(\preceq_1, ..., \preceq_I)$ satisfies Income Anonymity.*

The proof of this claim is in the appendix, along with other omitted proofs.

The second property is the standard (single-profile) Pareto condition:

Axiom (Pareto)**.** *For each $(\mathbf{m}, p), (\mathbf{m}', p') \in \mathbb{R}^I_{++} \times \mathcal{P}$,*

1. *If $(m_i, p) \preceq_i (m'_i, p')$ for all i, then $(\mathbf{m}, p) \preceq (\mathbf{m}', p')$; and*
2. *If, in addition, $(m_i, p) \prec_i (m'_i, p')$ for some i, then $(\mathbf{m}, p) \prec (\mathbf{m}', p')$.*

It is straightforward to see that if $\preceq^0$ are evaluation guidelines that satisfy multi-profile Pareto, then each ranking $\preceq^0$ $(\preceq_1, ..., \preceq_I)$ satisfies the single-profile Pareto condition.

The multi-profile setting therefore implies that, conditional on any preference profile, the evaluation guidelines for ranking of income distributions ought to satisfy both Income Anonymity and Pareto.

3 Characterization

On their own, both Pareto and Income Anonymity are consistent with many different social orderings. To illustrate, consider social welfare functions of the form

$$W(\mathbf{m}, p) = V(f_1(m_1, p), \ldots, f_I(m_I, p))$$

If $f_i = v_i$ for all i, then any strictly-increasing V would be compatible with Pareto. Similarly, if $f_i = f$ for all i, then any symmetric V would be compatible with Income Anonymity.

In addition, if all individuals have the same preferences (a case that is ruled out by Assumption 1), then there are many social preference relations that are consistent with both axioms. For example, when $f_i = v_i = v$ for all i, then both axioms are satisfied when V is a function that is strictly increasing and symmetric (and there are many such functions).[9]

The key result in this paper is that, whenever Assumption 1 is satisfied, then the combination of these axioms is consistent with at most one social preference relation. Furthermore,

[9]It is worth noting that, when preferences are identical, there are social orderings that are not representable by a social welfare function which also satisfy our two axioms. For example, the two axioms are satisfied by the Rawlsian social ordering, that cares lexicographically about the worst-off individual. Interestingly, Theorem 1 implies that when preferences are heterogeneous and satisfy Assumption 1, then our two axioms imply representability by a continuously differentiable social welfare function.

this social preference relation is representable by a continuously differentiable social welfare function that is additively-separable in individuals' money-metric utilities.

The following theorem states the main theoretical result of this paper.

Theorem 1. *If there exists a social ordering that satisfies Pareto and Income Anonymity, then it is unique. When it exists then it is represented by the social welfare function*

$$W_{i,i',j}(\mathbf{m},p)=\sum_{k=1}^{I}\int_{1}^{v_k(m_k,p)}\frac{dm'}{|\Delta x^j_{i,i'}(m')|} \tag{2}$$

for every $i\neq i'$ and j.

This theorem establishes that if there exists a social ordering that satisfies our axioms, then there is only one. When it exists, then it can be represented by $W_{i,i',j}$ *for any* $i\neq i'$ and j. In particular, the social welfare functions $\{W_{i,i',j}\}_{i,i',j}$ must all represent the same social ordering.

As we will see in Section 4, such a preference relation exists in some important special cases. However, there are also some preference profiles for which the two axioms are inconsistent. We return to this issue in Section 5.

When it exists, the social preference relation is unique and represented by (2); in particular, it must be (a) representable by a continuously differentiable social welfare function and (b) additively separable in money-metric utilities. These are desirable properties which we did not assume, but rather obtained as an implication of combining Pareto and Income Anonymity. Figure 1b illustrates how this social welfare function can be estimated based on the distribution of Engel curves, which is the basis of our estimation in Section 6.

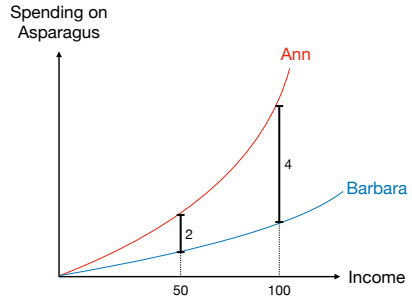
Below we sketch the key steps of the proof of uniqueness. This sketch also helps illustrate how $W_{i,i',j}$ is constructed, and the particular applications of the axioms that matter for its construction.

Before proceeding, it is useful to observe that the social welfare function in (2) implies that the ranking of income distributions conditional on the reference prices (p_0) is represented by

$$W_{i,i',j}(\mathbf{m},p_0)=\sum_{k=1}^{I}\int_{1}^{e_k(m_k,p_0,p_0)}\frac{1}{|\Delta x^j_{i,i'}(m')|}dm'=\sum_{k=1}^{I}\int_{1}^{m_k}\frac{1}{|\Delta x^j_{i,i'}(m')|}dm' \tag{3}$$

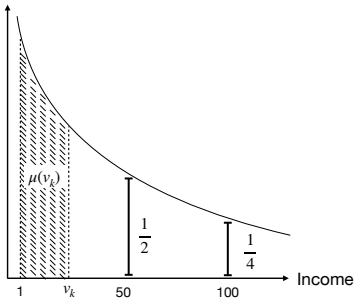
Furthermore, note that any $(\mathbf{m},p)$ is Pareto-equivalent to an allocation in which the prices are p_0 and incomes are equal to their money-metric utility levels $(((v_1(m_1,p),...,v_I(m_I,p)),p_0))$. As we are interested in characterizing a social ordering that satisfies Pareto, it is thus sufficient to show that $W_{i,i',j}(\cdot,p_0)$ is the only social ranking of allocations of the form $(\mathbf{m},p_0)$ that is consistent with our axioms.

Figure 1: Construction of the Social Welfare Function



(a) Engel curves

Note: Here, Ann is individual i, Barbara is individual i' and asparagus is good j. The function $\Delta x^j_{i,i'}(\cdot)$ is the difference between the two Engel curves. Here, $\Delta x^j_{i,i'}(50) = 2$ and $\Delta x^j_{i,i'}(100) = 4$.



(b) The social welfare function

Note: The social welfare function is given by the sum $W = \sum_{k=1}^{I} \mu(v_k(m_k, p))$, where $\mu(v_k) = \int_1^{v_k} dm/|\Delta x^j_{i,i'}(m)|$, and $\Delta x^j_{i,i'}(\cdot)$ is defined in Figure 1a based on the difference between Ann and Barbara's Engel curves.

The proof proceeds using the construction in Figure 2. Figure 2 presents the indifference curves of individuals i, i' and $k \neq i, i'$ over budgets that may differ in the price of good j, holding the prices of all other goods fixed at their reference levels. In this space, the individuals' indifference curves are upward sloping, because a higher price can be compensated with a higher income.

Consider an initial allocation in which prices are given by their reference level, p_0, and incomes are given by $\mathbf{m} = (m_1, ..., m_I)$. Construct an alternative price vector, p, for which $p^j = p_0^j + 1$ and $p^l = p_0^l$ for all $l \neq j$. It will be useful to think of "1" as a very small unit, so that the individuals' indifference curves are approximately linear between p_0 and p, and approximately parallel in small neighborhoods around their incomes. Define $\hat{\mathbf{m}} = (\hat{m}_1, ..., \hat{m}_I)$ based on the indifference conditions,

$$(m_l, p_0) \sim_l (\hat{m}_l, p) \text{ for all } l = 1, ..., I \tag{4}$$

Pareto requires that, when all individuals are indifferent, then the social preference relation is indifferent as well. Consequently, Pareto implies

$$(\mathbf{m}, p_0) \sim (\hat{\mathbf{m}}, p) \tag{5}$$

This first step is illustrated in Figure 2 by the three gray arrows labeled 1. Note that this step requires a limited application of the Pareto indifference condition, regarding small changes in the price of good j.

The second step uses Income Anonymity. Note that Income Anonymity requires the social ordering to be indifferent to any permutation of the income distribution at any prices. In particular, this implies that, given the allocation $(\hat{\mathbf{m}}, p)$, the social ordering ought to be indifferent to the permutation $\sigma_{i,i'}$, that switches the incomes of individuals i and i', leaving all other incomes in place:

$$(\hat{\mathbf{m}}, p) \sim (\sigma_{i,i'}(\hat{\mathbf{m}}), p) \tag{6}$$

This step is illustrated by the two gray arrows labeled 2.

The third step is an additional application of the Pareto indifference condition. Define $\tilde{\mathbf{m}}$ (or, equivalently, $\sigma_{i,i'}(\tilde{\mathbf{m}})$) based on the individual indifference conditions

$$(\hat{m}_{\sigma_{i,i'}(l)}, p) \sim_l (\tilde{m}_{\sigma_{i,i'}(l)}, p_0) \text{ for all } l = 1, ..., I$$

Using the Pareto indifference condition, we obtain

$$(\sigma_{i,i'}(\hat{\mathbf{m}}), p) \sim (\sigma_{i,i'}(\tilde{\mathbf{m}}), p_0) \tag{7}$$

This step is illustrated by the three gray arrows labeled 3.

The fourth step is an additional application of Income Anonymity, which implies

$$(\sigma_{i,i'}(\tilde{\mathbf{m}}), p_0) \sim (\sigma_{i,i'}^{-1}(\sigma_{i,i'}(\tilde{\mathbf{m}})), p_0) = (\tilde{\mathbf{m}}, p_0) \tag{8}$$

This step is illustrated by the two arrows labeled 4.

Finally, by the transitivity of the indifference relation, (5), (6), (7) and (8) imply

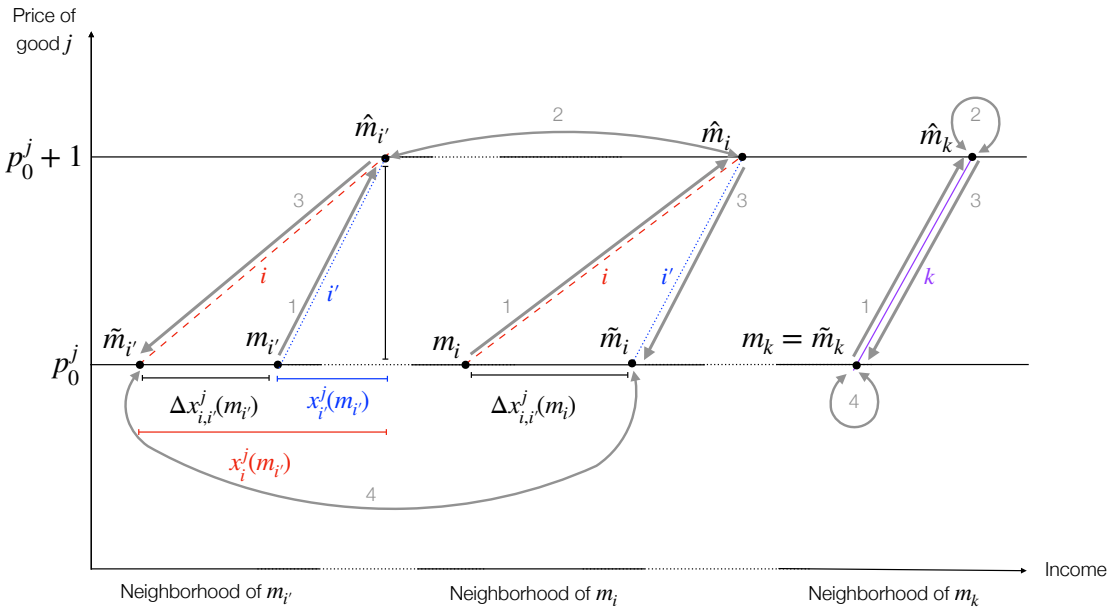
$$(\mathbf{m}, p_0) \sim (\tilde{\mathbf{m}}, p_0) \tag{9}$$

Consequently, at the reference prices, any social preferences that satisfy our axioms must be indifferent between the income distribution $\mathbf{m}$ and the income distribution $\tilde{\mathbf{m}}$. This means that, given the allocation $(\mathbf{m}, p_0)$, any social preference relation that satisfies our axioms must be indifferent with respect to simultaneously increasing m_i by $(\tilde{m}_i - m_i)$ and decreasing $m_{i'}$ by $(m_{i'} - \tilde{m}_{i'})$, leaving all other income levels unchanged (note that $m_k = \tilde{m}_k$ for all $k \neq i, i'$). This pins down the social marginal rate of substitution,

$$MRS(m_i, m_{i'}|\mathbf{m}, p_0) = \frac{m_{i'} - \tilde{m}_{i'}}{\tilde{m}_i - m_i} \tag{10}$$

Since $\mathbf{m}$ was chosen arbitrarily, a similar construction pins down all of the social marginal rates of substitution between the incomes of individuals i and i', depending on their income levels. Furthermore, this construction relates these marginal rates of substitution to the Engel curves of individuals i and i'. To see this, note that, in Figure 2, it is apparent that

Figure 2: Sketch of the proof of Theorem 1



individual i cares more about the price of good j, because he demands a higher monetary compensation for the increase in its price. Intuitively, the extent to which an individual cares about a good's price depends on the quantity of the good that he consumes. It turns out (using Roy's identity) that the amount of compensation required is approximately equal to the individual's consumption of good j. Using this approximation, we obtain

$$m_{i'} - \tilde{m}_{i'} \approx x^j_i(m_{i'}) - x^j_{i'}(m_{i'}) = \Delta x^j_{i,i'}(m_{i'}) \text{ and}$$
$$\tilde{m}_i - m_i \approx x^j_i(m_i) - x^j_{i'}(m_i) = \Delta x^j_{i,i'}(m_i)$$

Hence, the social marginal rates of substitution are given by

$$MRS(m_i, m_{i'}|\mathbf{m}, p_0) = \frac{m_{i'} - \tilde{m}_{i'}}{\tilde{m}_i - m_i} = \frac{\Delta x^j_{i,i'}(m_{i'})}{\Delta x^j_{i,i'}(m_i)}$$

The proof concludes by using Income Anonymity to show that this formula must also characterize the marginal rates of substitution between m_k and $m_{k'}$ for every two individuals, k and k':

$$MRS(m_k, m_{k'}|\mathbf{m}, p_0) = \frac{\Delta x^j_{i,i'}(m_{k'})}{\Delta x^j_{i,i'}(m_k)} \tag{11}$$

Thus, this construction fully characterizes the social marginal rates of substitution condi-

tional on the reference prices. As an ordering is uniquely characterized by its marginal rates of substitution, the uniqueness result follows. Using expression (3), it is easy to show that these are the marginal rates of substitution implied by the social ranking represented by $W_{i,i',j}(\cdot, p_0)$.

4 Examples

In this section, we focus on four common classes of individual preferences. In each of these examples, the social orderings represented by (2) are the same for all i, i' and j, and correspond to the unique ordering that satisfies our axioms. Homothetic preferences are defined over all budgets in $\mathbb{R}_{++} \times \mathcal{P}$. The remaining three classes are defined only for incomes above a price-dependent threshold (for example, Stone-Geary preferences are defined only for budgets in which the subsistence bundle is affordable). Our results also apply on such restricted domains; Appendix D provides the details, including the specific domain for each example.

Homothetic preferences. When preferences are homothetic, the money-metric utility functions satisfy $v_i(m, p) = m v_i(1, p)$ for every m. Consider the social welfare function

$$W(\mathbf{m}, p) = \sum_{i=1}^{I} \ln(v_i(m_i, p)) = \sum_{i=1}^{I} \ln(m_i v_i(1, p)) = \sum_{i=1}^{I} \ln(m_i) + \sum_{i=1}^{I} \ln(v_i(1, p)) \tag{12}$$

This social welfare function is Paretian, because it is a sum of individual utility functions ($\ln(v_i(m_i, p))$). It satisfies Income Anonymity because it is symmetric in $m_1, ..., m_I$. By Theorem 1, it follows that it represents the unique ordering that satisfies both axioms.

Conditional on prices, the social preference relation represented by (12) ranks income distributions according to the sum of log incomes. Note that we did not assume anywhere that individual preferences are logarithmic - individuals may have arbitrary constant-elasticity-of-substitution preferences, or other homothetic preferences.[10] Regardless, the social ranking will rank income distributions according to their geometric means.

Stone-Geary preferences. Consider a modification of the homothetic case, in which preferences are non-homothetic due to the presence of a common subsistence bundle. To survive, each individual must consume at least $\underline{x}^j \geq 0$ goods of type j. However, after

[10] CES preferences with an elasticity of substitution in the $(0, 1]$ range violate strict convexity on the boundary of $\mathbb{R}^J_+$. Our analysis nevertheless applies to these preferences. Because prices are strictly positive, any budget with positive income allows the individual to afford a strictly positive bundle, which is strictly preferred to any bundle on the boundary. Hence, the optimal bundle is interior, where these preferences are strictly convex, so that $x_i(m, p)$ is uniquely defined.

consuming the subsistence bundle, individuals' preferences over their remaining consumption are homothetic; formally, there exist utility functions, $\{H_i\}_{i=1}^I$, which are homogeneous of degree 1, such that individual preferences are represented by $H_i(x^1 - \underline{x}^1, ..., x^J - \underline{x}^J)$.

Define the homothetic indirect preferences associated with H_i as

$$\tilde{v}_i(m,p) = \max_x H_i(x) \text{ s.t. } \sum_{j=1}^J p^j x^j \leq m$$

As H_i is homogeneous of degree 1, it follows that $\tilde{v}_i(m,p) = m\tilde{v}_i(1,p)$. Note that $\tilde{v}_i(m,p)$ does not represent the indirect preferences of individual i, because it does not take into account the cost of the subsistence bundle. Rather, the indirect preferences of individual i are represented by $\tilde{v}_i(m - \sum_{j=1}^J p^j \underline{x}^j, p)$ because, after buying the subsistence bundle, the remaining income is used for maximizing H_i.

Consider the social welfare function

$$\begin{aligned} W(\mathbf{m},p) &= \sum_{i=1}^I \ln(\tilde{v}_i(m_i - \sum_{j=1}^J p^j \underline{x}^j, p)) = \sum_{i=1}^I \ln((m_i - \sum_{j=1}^J p^j \underline{x}^j)\tilde{v}_i(1,p)) \\ &= \sum_{i=1}^I \ln(m_i - \sum_{j=1}^J p^j \underline{x}^j) + \sum_{i=1}^I \ln(\tilde{v}_i(1,p)) \end{aligned}$$

This social welfare function is Paretian and symmetric in $m_1, ..., m_I$. Hence, it represents the unique ordering that satisfies our axioms.

Note that, in this case, the social ranking of income distributions depends on the price vector.[11] In particular, inequality aversion is higher when the subsistence bundle is more expensive.

Gorman form preferences. Assume that individual preferences are of the Gorman polar form, so that each $\preceq_i$ is represented by

$$g(p)m + \gamma_i(p) \tag{13}$$

As Gorman [1961] shows, this condition is necessary and sufficient for deriving an aggregate demand function that depends only on aggregate income, rather than on the distribution of income. A prominent example of such preferences are quasilinear preferences of the form

$$u_i(x^1, ..., x^J) = x^1 + f_i(x^2, ..., x^J)$$

[11] This illustrates that our social welfare function does not always satisfy Roberts' "Price Independent Welfare Prescriptions" axiom [Roberts, 1980].

where f_i is strictly concave. Note that the functions $\{f_i\}_{i=1}^I$ may be heterogeneous and take arbitrary functional forms. Consider the sum of indirect utility functions,

$$W(\mathbf{m}, p) = \sum_{i=1}^{I} (g(p)m_i + \gamma_i(p)) \tag{14}$$

This social welfare function is Paretian and symmetric in $m_1, ..., m_I$. Hence, it represents the unique ordering that satisfies our axioms. Unlike the previous examples, this preference ranking exhibits no aversion to income inequality.

Price independent generalized linearity (PIGL) preferences. Muellbauer [1975] and Muellbauer [1976] define PIGL preferences as preferences that can be represented by indirect utility functions of the form

$$\frac{1}{\epsilon}\left(\frac{m}{h(p)}\right)^{\epsilon} - \gamma_i(p)$$

where h is homogeneous of degree 1 and γ_i is homogeneous of degree 0. These preferences have proven useful for studying non-homotheticity in the context of long-run economic growth (see, for example, Boppart [2014] and Fan et al. [2023]). The parameter $\epsilon \in (0, 1)$ is often referred to as the "Engel elasticity", as it captures the extent to which expenditure shares on different goods vary with income. Theorem 1 implies that, given such preferences, there exists a unique social ordering that satisfies Pareto and Income Anonymity. This social ordering is represented by the sum of indirect utility functions,

$$W(\mathbf{m}, p) = \sum_{i=1}^{I} \left(\frac{1}{\epsilon}\left(\frac{m_i}{h(p)}\right)^{\epsilon} - \gamma_i(p)\right)$$

(to see this, note that this social welfare function trivially satisfies both axioms).

In this class of models, the ranking of income distributions is independent of prices, as noted by Roberts [1980]. This ranking corresponds to an Atkinson index with an inequality aversion parameter of $1 - \epsilon$.[12]

5 Existence conditions

Consider the following theorem, which lays out necessary and sufficient conditions for the consistency of Pareto and Income Anonymity.

[12] See (15) for an explicit definition of the inequality aversion parameter in this class of models.

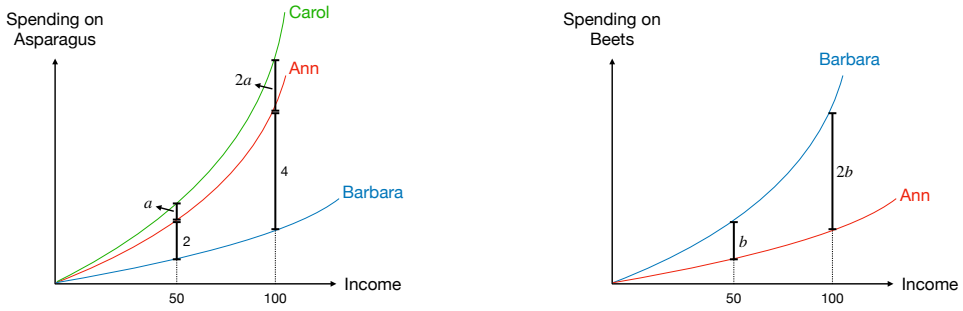
Theorem 2. *There exists a social ordering that satisfies Pareto and Income Anonymity if and only if the following two conditions hold:*

1. ***The proportionality condition:*** *For every i, i' and j, there exists some $\alpha_{i,i',j}$ such that $\Delta x^j_{i,i'}(\cdot) = \alpha_{i,i',j}\Delta x^1_{1,2}(\cdot)$.*

2. ***The separability condition:*** *Each individual preference ranking, $\preceq_i$, has a representation of the form $f_i(v_1(m,p))+g_i(p)$ where f_i and g_i are continuously differentiable.*

The proportionality condition guarantees that the social welfare functions $\{W_{i,i',j}\}$ (as defined in (2)) are identical up to a multiplication by a constant. As a result, they all represent the same ranking.

Figure 3: The proportionality condition



Note: All differences between Engel curves must be proportional to one another.

This condition can be easily verified based on the distribution of Engel curves, as illustrated in figure 3. This figure illustrates that the proportionality condition is already quite restrictive: for example, a small perturbation to Ann's demand for asparagus could lead to its violation.[13]

The second condition is an additive separability condition. This condition is used for guaranteeing that the rankings represented by $\{W_{i,i',j}\}$ remain symmetric in incomes even away from the reference prices. Theorem III.6.6 in Wakker [1989] clarifies the restrictions that additive separability places on the preference profile.

To illustrate the implications of this condition, assume that individual k's preferences are represented by $f_k(v_1(m,p)) + g_k(p)$, and consider the comparison between two budgets,

[13] A simple example of an economy that violates this proportionality condition is one with two individuals and three goods: Individual 1 consumes only good 1, and individual 2 has quasi-linear preferences over goods 2 and 3, with good 2 being the numeraire good (so that, at the prices p_0, he consumes a constant amount, c, of good 3). In this case, we have that $\Delta x^1_{1,2}(m) = m$ and $\Delta x^3_{1,2}(m) = -c$. Since $\Delta x^1_{1,2}(\cdot)$ is not proportional to $\Delta x^3_{1,2}(\cdot)$, this is a violation of the proportionality condition.

(m, p) and (m', p'). Assume that individual 1 is indifferent between these two budgets, so that $v_1(m, p) = v_1(m', p')$. Using the additively-separable representation, we can say that the move from (m, p) to (m', p') will lead to a loss of "10 utility units" for individual k if

$$(f_k(v_1(m, p)) + g_k(p)) - (f_k(v_1(m', p')) + g_k(p')) = g_k(p) - g_k(p') = 10$$

The additive separability of this utility function means that this utility loss would be the same regardless of the level of $v_1(m, p)$. Specifically, whenever we change prices from p to p' and adjust individual k's income in a way that would have made *individual 1* indifferent, the resulting utility loss for individual k would be 10, where the units of utility are specified in terms of levels of this additively-separable representation.

Of course, these utility units themselves are meaningless: we can always multiply f_k and g_k by a positive constant, $a > 0$, and obtain a different additively-separable representation according to which the utility loss would have been $10a$ instead of 10. However, what matters is that there exists some measuring rod (not necessarily unique) according to which the utility loss does not depend on the level of $v_1(m, p)$: as long as income changes in a way that would exactly compensate individual 1, the utility loss for individual k depends only on the price change.

In the examples in Section 4, this condition is satisfied. However, it is also possible to construct preference profiles in which this condition is violated, and there is no reason to expect it to hold in general.[14]

Theorem 2 therefore establishes that, while our axioms are consistent in many canonical examples, there are also cases in which they are in conflict. In such cases, the social welfare functions in (2) may represent different rankings depending on i, i' and j, and none of them will satisfy Income Anonymity.[15]

Nonetheless, this family of social welfare criteria has an intuitive normative interpretation. As evident from the construction in Figure 2, the marginal rates of substitution of $W_{i,i',j}$ conditional on the reference prices, p_0, are implied by the combination of three assumptions. The first is an application of the Pareto indifference conditions in scenarios involving small changes in the price of good j (steps 1 and 3 in Figure 2). This is a restricted application of the Pareto condition to small changes in the price of good j around the reference prices. The second assumption is a limited application of Income Anonymity which requires that, when the price of good j deviates slightly from its reference level, society is indifferent with respect to switching the incomes of i and i' (step 2 in Figure 2). The final condition is that

[14] Appendix E.1 shows an example in which this condition is violated.

[15] Note that as $W_{i,i',j}$ is a strictly increasing function of individual money-metric utilities, $v_1(m_1, p), ..., v_I(m_I, p)$, it is always consistent with Pareto.

the social ordering should be indifferent to any permutation of the income distribution at the reference prices (step 4 in Figure 2).[16]

Thus, the family of social welfare criteria represented by $\{W_{i,i',j}\}$ is uniquely characterized by different ways of weakening our two axioms, Pareto and Income Anonymity. While there may be other ways of weakening these axioms, this family of criteria emerges naturally from our characterization.

6 Application

We now turn to a calibration of the social welfare functions identified in (2) based on consumer expenditure data. Our goal is to provide a practical theory-grounded approach for decision making based on our results. We find that, for US consumers, our social welfare functions rank income distributions approximately according to the sum of log incomes.

Our objective is to calibrate $W_{i,i',j}(\cdot, p_0)$ rather than the entire ranking, $W_{i,i',j}(\cdot,\cdot)$. In particular, using consumer expenditure surveys, we calibrate (3), where p_0 is the vector of prices that surveyed consumers face (ignoring possible price heterogeneity). The output of our procedure is a ranking of income distributions conditional on prices, rather than a complete ranking of all possible combinations of income distributions and prices.

6.1 Calibration procedure

To calculate expression (3), we must know the demand functions, $x_i^j(\cdot)$ and $x_{i'}^j(\cdot)$. Unfortunately, surveys typically tell us only what people consume given their actual budgets, and not what they would consume given hypothetical budgets. To proceed, we need to relate demand functions to the cross-sectional distribution of expenditures.

Let $x_r^j(m)$ denote the r-th percentile of the distribution of expenditures on good j, given income m and prices p_0. For consistency of notation, we normalize $p_0^j = 1$ for every j, so that $x_r^j(m)$ is also the quantity of good j consumed. Consider the following assumption.

Assumption 2. *For each i, j and m, if $x_i^j(m) = x_r^j(m)$ for some percentile r, then, for every m', it holds that $x_i^j(m') = x_r^j(m')$.*

This assumption says that if Ann's consumption of asparagus is at the 75th percentile among all people of her income level, then this would be true regardless of what her income level happens to be. This assumption attributes any differences between consumption patterns between rich and poor people to differences in their incomes, rather than differences in

[16] This last assumption is also used for obtaining social marginal rates of substitution that depend only on individuals' income levels, and not on their identities (expression (11)).

their preferences. It does not allow for the possibility that historical or other factors affect both income and tastes.

Under this assumption, we can identify "individual i" as an individual whose consumption of good j is at the $\bar{r}$-th percentile among others with the same income level, and "individual i'" as an individual whose consumption is at the $\underline{r}$-th percentile among others with the same income level. We can empirically estimate the functions $x_i^j(\cdot) = x_{\bar{r}}^j(\cdot)$ and $x_{i'}^j(\cdot) = x_{\underline{r}}^j(\cdot)$, based on how the relevant percentiles of the consumption of good j change with income. We can then define the function $\mu(\cdot)$ as

$$\mu(m) := \int_1^m \frac{1}{x_{\bar{r}}^j(m') - x_{\underline{r}}^j(m')} dm'.$$

Our estimated social welfare function is then

$$W_{i,i',j}(\mathbf{m}, p_0) = \sum_{k=1}^{I} \mu(m_k)$$

Note that this construction depends on several choices. First, we need to choose an expenditure category (food, housing, etc.)— this corresponds to choosing which good plays the role of "good j". Second, we need to choose which two percentiles to compare ($\underline{r}$ and $\bar{r}$). This corresponds to choosing which individuals play the roles of "individual i" and "individual i'". As we will see, different specifications indeed yield somewhat different estimates of μ. However, it turns out that the results are not very sensitive to these choices.

6.2 Empirical implementation

Our data source is the United States Consumer Expenditure Survey's Quarterly Interview. It has been conducted quarterly since 1980, and each survey contains a (weighted) representative sample of approximately 6,000 US households. Households are asked to report their expenditures on 14 expenditure categories over the previous quarter.

We use data for the four quarters between 2021-Q4 and 2022-Q3. We perform two adjustments to the sample. First, we restrict our sample to single consumer units. This sample restriction allows us to circumvent well-known difficulties with comparing consumption across different family structures. Second, we drop the bottom 5% and the top 10% of expenditure levels.[17] The purpose of this sample restriction is to get rid of atypical consumption quarters. If a household happens to be interviewed in a quarter in which it purchased a house or a car, its reported expenditure will vastly exceed its typical consumption expenditure.

[17] Dropping the bottom 5% is sufficient for removing all negative and zero values.

Similarly, there may be some quarters with unusually low expenditures.[18] Figure 5 in the appendix shows the distribution of total consumption expenditure without removing the extreme values.

The number of observations in our final sample is 5549 consumption units. Descriptive statistics for each expenditure category for this sample are provided in Table 1 in the appendix.

In our theoretical framework, m represents both income and consumption expenditure (equation (1)). In practice, income varies over the lifecycle, and people can smooth these fluctuations through borrowing and saving. It is therefore more appropriate to identify m with lifetime income. Here, we estimate m using quarterly consumption expenditure, echoing the view that people smooth consumption over time.[19]

We chose not to supplement consumption expenditure with an imputed value of leisure or home production. A person's leisure at a given quarter is a poor measure of his lifetime leisure consumption. As income varies over the lifecycle, so does leisure. Most notably, people consume vastly more leisure after retirement. From a theoretical perspective, we can ignore the value of leisure if we are willing to assume that labor supply is inelastic. This assumption is problematic for married couples that may choose to have one spouse specialize in home production (for example, a stay-at-home parent). However, as our sample contains only single consumer units, the avenues for substituting home production for market production seem more limited.

We estimate $W_{i,i',j}$ for multiple consumption categories, as well as multiple combinations of high and low consumption percentiles. We use five different consumption categories: housing, food, transport, health, and entertainment. These are the consumption categories with the largest means in the dataset, and the only ones with a majority of nonzero observations.[20] Our choice of consumption percentiles is arbitrary, and aimed to reflect a range of possibilities. The (low/high) percentile pairs we chose for our estimations were: 25/75, 10/90, 10/60, and 40/90.

For estimating the Engel curves, $x_r^j(\cdot)$, we proceed as follows. We re-weigh our sample so that weights are distributed equally across the expenditure distribution, and estimate the coefficients of the polynomial approximation $x_r^j(m) = \sum_{n=0}^{5} \beta_n m^n$. This estimation is done

[18] For example, because insurance reimbursements are recorded as negative expenses, total expenditures may be abnormally small or even negative in quarters in which the household is reimbursed for an expensive medical procedure.

[19] Household expenditure includes some categories which are not obviously "consumption" categories, such as education, cash contributions and insurance and pension contributions. Our results are robust to the exclusion of these categories from our measure of m.

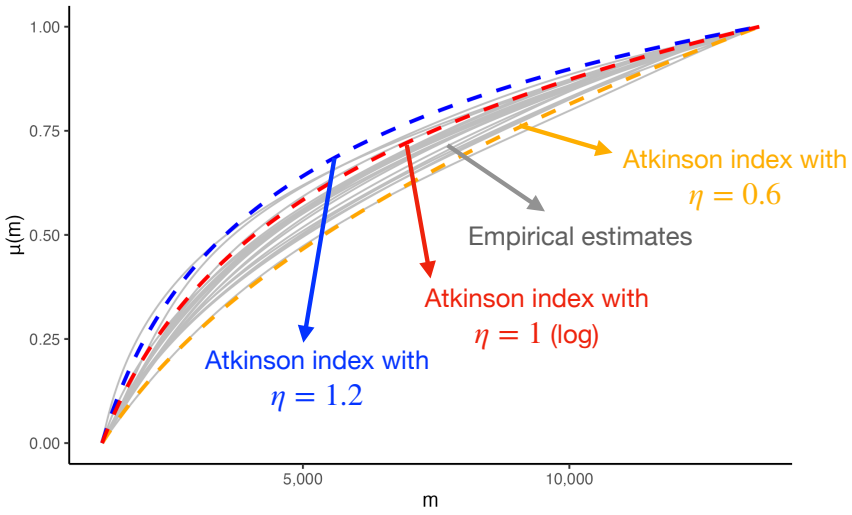
[20] Even though insurance and pensions is a larger category than entertainment, we exclude it because it is more appropriately classified as savings than consumption.

through a quantile regression for each of the percentiles we use. We choose a polynomial of degree 5, which provides a good fit for the data (see Figure 6 in the appendix). Our results are robust to variations in the polynomial degree.

6.3 Results

Figure 4 presents the estimates obtained from our various specifications. As expected, calibrations based on different Engel curves yield somewhat different estimates of μ. Nevertheless, the results illustrate a surprising uniformity of the estimates across the various specifications.

Figure 4: Empirical estimates of μ



Note: Each of the 20 gray lines represents the estimated μ for a combination of expenditure category and a pair of expenditure quantiles. Dashed lines represent the Atkinson index with different inequality aversion coefficients. All lines are normalized so that their range is between zero and one in the given domain of total expenditure values.

The concavity of μ reflects the degree of inequality aversion. To interpret this concavity, we compare our estimated social welfare functions to various instances of the Atkinson Index. The Atkinson index [Atkinson, 1970] is a ranking of income distributions (conditional on prices) that is represented by

$$W^{Atkinson}(\mathbf{m}) = \frac{1}{1-\eta}\left(\sum_{i=1}^{I} m_i^{1-\eta} - I\right) \tag{15}$$

where η is a parameter that governs the degree of inequality aversion. The limit $\eta \to$

1 corresponds to the log case, as in the example of homothetic preferences. A higher η corresponds to a social ranking that is more inequality averse.

Across the different specifications, the curvature of μ lies between the curvature of an Atkinson index with $\eta = 0.6$ and the curvature of an Atkinson index with $\eta = 1.2$. This places some bounds on the degree of inequality aversion implied by our welfare criteria.

Our finding is based on the empirical regularity that the distribution of consumption expenditures is roughly proportional to income. We would obtain lower degrees of inequality aversion if we found that consumption patterns become more similar at higher income levels, and higher degrees of inequality aversion if we found that consumption becomes more heterogeneous at higher income levels.

7 Conclusion

Income Anonymity is a formulation of the idea that we should be able to rank income distributions without knowing which income belongs to which person. We have shown that, in some canonical examples, the combination of this principle with Pareto implies concrete guidelines for how income distributions should be ranked.

In some cases, the two axioms are inconsistent. Nonetheless, our characterization identifies a family of welfare criteria that corresponds to different ways of weakening the anonymity axiom, in the neighborhood of some reference prices. Under some assumptions, these criteria can be evaluated based on the cross-sectional distribution of consumption expenditures. We find that, in the particular case of recent consumption data in the United States, our axioms imply a social objective that is approximately the maximization of the sum of log incomes.

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A Informational restrictions

This appendix contains a formalization of the informational restrictions embedded in the Independence of Irrelevant Prices axiom.

Consider a model with the following timeline:

1. A decision maker knows the statistical distribution of preferences, but not which preferences belong to which individual. In particular, if the true preference profile is $(\preceq_1, ..., \preceq_I)$, he knows only that the preference profile is $(\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})$ for some permutation σ. He cannot rule out any σ and considers any permutation to be equally likely.

2. The decision maker is tasked with specifying *decision guidelines* (as opposed to evaluation guidelines). Decision guidelines are a function $C : \mathcal{F} \times R^I \to \mathbb{R}^I_{++} \times \mathcal{P}$, where $\mathcal{F}$ is a domain of policy questions ($F \in \mathcal{F}$ is a set of feasible budget allocations that the policymaker must choose from), and $(r_1, ..., r_I) \in R^I$ are individuals' *reported* preferences. Each $r_i \in R$ is a ranking of individual budgets, and R is the domain of possible reports, which is given by $R = \{\preceq_1, ..., \preceq_I\}$ (this domain restriction reflects the fact that the decision maker knows the statistical distribution of preferences – this means that these are the only individual preference rankings that cannot be immediately ruled out as false). For each $F \in \mathcal{F}$, it must hold that $C(F, (r_1, ..., r_I)) \in F$.

3. After the decision guidelines are specified, a policy question, $F \in \mathcal{F}$, is revealed.

4. Knowing the decision guidelines and the policy question, each individual reports a preference ranking, $r_i \in R$.

5. The chosen policy is $C(F, (r_1, ..., r_I))$.

Consider the following (strong) incentive compatibility requirement:

Axiom (Incentive Compatibility)**.** *For every i and $(r_1, ..., r_I), (\tilde{r}_1, ..., \tilde{r}_I) \in R^I$,*

$$[C(F, (\tilde{r}_1, ..., \tilde{r}_I))]_i \preceq_{r_i} [C(F, (r_1, ..., r_I))]_i$$

where $[C(F, r)]_i$ is the budget allocated to individual i given the choice $C(F, r)$.

This condition states that no individual ever has an incentive to lie about her preferences, regardless of what others report, and regardless of whether others would change their report depending on her report. Specifically, if individual i's true preferences are r_i, then it is always best for her to truthfully report her preferences as r_i, regardless of what others decide to do.

Consider the following axiom on C, which relates the evaluation guidelines to the decision guidelines.

Axiom (Rationality)**.** *For each permutation, σ, there exists some social ranking, $\preceq^0$ $(\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})$, such that for each $F \in \mathcal{F}$, it holds that*

$$(\mathbf{m}, p) \preceq^0 (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)}) C(F, (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})) \text{ for all } (\mathbf{m}, p) \in F \tag{16}$$

This axiom states that the decision guidelines must correspond to the maximization of some social objective. This social objective constitutes the evaluation guidelines.

The following claim establishes that Rationality and Incentive Compatibility imply Independence of Irrelevant Prices.

Claim 2. *Assume that C satisfies Incentive Compatibility, and let $\preceq^0$ $(\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})$ be consistent with (16). Then, the evaluation guidelines $\preceq^0$ satisfy Independence of Irrelevant Prices.*

Proof. Fix some p and let $F = \{(\mathbf{m}, p), (\mathbf{m}', p)\}$ where $\mathbf{m} \neq \mathbf{m}'$. We want to show that $C(F, (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})) = C(F, (\preceq_1, ..., \preceq_I))$ for every σ. Assume by way of contradiction that this equality doesn't hold; without loss of generality, assume that, for some σ,

$$C(F, (\preceq_1, ..., \preceq_I)) = (\mathbf{m}, p) \text{ and } C(F, (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)})) = (\mathbf{m}', p)$$

By Incentive Compatibility, for every i, it holds that

$$[C(F, (\preceq_{\sigma(1)}, ..., \preceq_{\sigma(I)}))]_i \preceq_i [C(F, (\preceq_1, ..., \preceq_I))]_i$$

As both alternatives have the same price vector, this implies that

$$m'_i \leq m_i \quad \forall i \tag{17}$$

Similarly, using Incentive Compatibility, we have that

$$[C(F,(\preceq_1,...,\preceq_I))]_i \preceq_{\sigma(i)} [C(F,(\preceq_{\sigma(1)},...,\preceq_{\sigma(I)}))]_i$$

which then similarly implies that

$$m_i \leq m'_i \quad \forall i \tag{18}$$

Combining (18) and (17) implies that $m_i = m'_i$ for all i, in contradiction to the assumption that $\mathbf{m} \neq \mathbf{m}'$. Thus, we obtain $C(F,(\preceq_{\sigma(1)},...,\preceq_{\sigma(I)})) = C(F,(\preceq_1,...,\preceq_I))$.

Hence, using (16), it follows that

$$(\mathbf{m},p) \preceq^0 (\preceq_{\sigma(1)},...,\preceq_{\sigma(I)})(\mathbf{m}',p) \Leftrightarrow (\mathbf{m},p) \preceq^0 (\preceq_1,...,\preceq_I)(\mathbf{m}',p)$$

as required by Independence of Irrelevant Prices. □

The above claim therefore establishes Independence of Irrelevant Prices as a result of incentive compatibility constraints. It is also worth highlighting that these incentive compatibility constraints would indeed be satisfied by decision guidelines that are based on the social welfare function in (2) whenever our axioms are consistent. This follows from Theorem 2: when the axioms are consistent, then each $\preceq_i$ can be represented by $f(v_1(m,p))+\tilde{g}_i(p)$ (see (33)). In this case, the social ordering given by

$$W(m,p) = \sum_{i=1}^{I} (f(v_1(m_i,p)) + \tilde{g}_i(p)) = \sum_{i=1}^{I} f(v_1(m_i,p)) + \sum_{i=1}^{I} \tilde{g}_i(p)$$

satisfies both Pareto and Income Anonymity, and hence it represents the same social ranking as (2). Hence, we have that each $\preceq^0 (\preceq_{\sigma(1)},...,\preceq_{\sigma(I)})$ is represented by

$$\begin{aligned} W_\sigma(m,p) &= \sum_{i=1}^{I} \left(f(v_1(m_i,p)) + \tilde{g}_{\sigma(i)}(p)\right) = \sum_{i=1}^{I} f(v_1(m_i,p)) + \sum_{i=1}^{I} \tilde{g}_{\sigma(i)}(p) \\ &= \sum_{i=1}^{I} f(v_1(m_i,p)) + \sum_{i=1}^{I} \tilde{g}_i(p) = W(m,p) \end{aligned}$$

Thus, the evaluation guidelines conditional on $(\preceq_{\sigma(1)},...,\preceq_{\sigma(I)})$ do not depend on σ. As the evaluation guidelines are designed by a decision maker who already knows the distribution of preferences, they will be the same regardless of individuals' reports. Consequently, individuals have no incentive to misreport their preferences, as the evaluation guidelines will not depend on their reports in any way.

B Proof of Claim 1

Fix some permutation, σ. Note that σ can be written as a composition of swaps, $\sigma = \sigma_N \circ \cdots \circ \sigma_1$, where each σ_n is a permutation that swaps two indexes and leaves the rest constant (for example, $(1,2,3,4) \mapsto (3,2,1,4)$ is a permutation that swaps the first and third indexes, leaving the second and the fourth constant). Observe that, for each σ_n that is a swap, it holds that $\sigma_n \circ \sigma_n$ is the identity. Thus, σ_n is its own inverse: $\sigma_n^{-1} = \sigma_n$.

First, we show that, for each swap σ_n it holds that

$$(\mathbf{m}, p) \sim^0 (\preceq_1, ..., \preceq_I)(\sigma_n(\mathbf{m}), p) \text{ for all } \mathbf{m}, p$$

where $\sim^0 (\preceq_1, ..., \preceq_I)$ is the indifference relation corresponding to the ranking $\preceq^0 (\preceq_1, ..., \preceq_I)$.

To see this, note that, by Independence of Irrelevant Prices,

$$(\mathbf{m}, p) \preceq^0 (\preceq_1, ..., \preceq_I)(\sigma_n(\mathbf{m}), p) \Leftrightarrow (\mathbf{m}, p) \preceq^0 (\preceq_{\sigma_n(1)}, ..., \preceq_{\sigma_n(I)})(\sigma_n(\mathbf{m}), p)$$

By Classical Anonymity,

$$\begin{aligned}(\mathbf{m}, p) \preceq^0 (\preceq_{\sigma_n(1)}, ..., \preceq_{\sigma_n(I)})(\sigma_n(\mathbf{m}), p) &\Leftrightarrow (\sigma_n^{-1}(\mathbf{m}), p) \preceq^0 (\preceq_1, ..., \preceq_I)(\sigma_n^{-1}(\sigma_n(\mathbf{m})), p) \\ &\Leftrightarrow (\sigma_n(\mathbf{m}), p) \preceq^0 (\preceq_1, ..., \preceq_I)(\mathbf{m}, p)\end{aligned}$$

(where the last step uses the identity $\sigma_n^{-1} = \sigma_n$).

Combining, we have that

$$(\mathbf{m}, p) \preceq^0 (\preceq_1, ..., \preceq_I)(\sigma_n(\mathbf{m}), p) \Leftrightarrow (\sigma_n(\mathbf{m}), p) \preceq^0 (\preceq_1, ..., \preceq_I)(\mathbf{m}, p)$$

This implies that

$$(\mathbf{m}, p) \sim^0 (\preceq_1, ..., \preceq_I)(\sigma_n(\mathbf{m}), p)$$

as required.

As we have shown that this holds for each σ_n, this must also hold for the composition $\sigma = \sigma_N \circ ... \circ \sigma_1$:

$$\begin{aligned}(\mathbf{m}, p) &\sim^0 (\preceq_1, ..., \preceq_I)(\sigma_1(\mathbf{m}), p) \sim^0 (\preceq_1, ..., \preceq_I)(\sigma_2(\sigma_1(\mathbf{m})), p) \\ &\sim^0 (\preceq_1, ..., \preceq_I)(\sigma_N(\ldots(\sigma_1(\mathbf{m}))), p) = (\sigma(\mathbf{m}), p)\end{aligned}$$

C Proof of Theorem 1

Consider the following claims, which will be used in the proof of the theorem.

Claim 3. *For every k, m, at $p = p_0$, it holds that* $\frac{\partial e_k(m,p,p)}{\partial p_T^j} = -\frac{\partial e_k(m,p,p)}{\partial p_S^j} = x_k^j(m,p)$.

Proof. For every k and m,

$$v_k(m, p_S) = v_k(e_k(m, p_S, p_T), p_T) \tag{19}$$

(by the definition of e_k). Taking the derivative with respect to p_T^j (using the assumed differentiability of e_k),

$$\frac{\partial v_k(m,p_S)}{\partial p_T^j} = \frac{\partial v_k(e_k(m,p_S,p_T),p_T)}{\partial p_T^j} = \frac{\partial v_k(\bar{e},p_T)}{\partial \bar{e}}\frac{\partial e_k(m,p_S,p_T)}{\partial p_T^j} + \frac{\partial v_k(\bar{e},p_T)}{\partial p_T^j}$$

where $\bar{e} = e_k(m, p_S, p_T)$. As the left hand side is 0, this implies that

$$\frac{\partial e_k(m,p_S,p_T)}{\partial p_T^j} = -\frac{\frac{\partial v_k(\bar{e},p_T)}{\partial p_T^j}}{\frac{\partial v_k(\bar{e},p_T)}{\partial \bar{e}}} = x_k^j(\bar{e},p_T)$$

where the last step follows from Roy's identity. Note that, when $p_S = p_T$, $\bar{e} = m$; hence, it follows that

$$\frac{\partial e_k(m,p,p)}{\partial p_T^j} = x_k^j(m,p)$$

Taking the derivative of (19) with respect to p_S^j, we have

$$\frac{\partial v_k(m,p_S)}{\partial p_S^j} = \frac{\partial v_k(e_k(m,p_S,p_T),p_T)}{\partial p_S^j} = \frac{\partial v_k(\bar{e},p_T)}{\partial \bar{e}}\frac{\partial e_k(m,p_S,p_T)}{\partial p_S^j}$$

Rearranging, it follows that

$$\frac{\partial e_k(m,p_S,p_T)}{\partial p_S^j} = \frac{\frac{\partial v_k(m,p_S)}{\partial p_S^j}}{\frac{\partial v_k(\bar{e},p_T)}{\partial \bar{e}}}$$

Note that when $p_S = p_T = p$, we have that $\bar{e} = e_k(m, p, p) = m$. Thus,

$$\frac{\partial e_k(m,p,p)}{\partial p_S^j} = \frac{\frac{\partial v_k(m,p)}{\partial p^j}}{\frac{\partial v_k(m,p)}{\partial m}} = -x_k^j(m,p)$$

where the last step again follows from Roy's identity. □

The proof of the theorem follows the steps outlined in Figure 2. Fix some $i \neq i'$ and j. Assume that the social ordering satisfies Pareto and Income Anonymity. We begin by showing that it is unique and represented by (2).

It is convenient to introduce the notation

$$e(\mathbf{m}, p_S, p_T) = (e_1(m_1, p_S, p_T), ..., e_I(m_I, p_S, p_T))$$

Denote $p(\epsilon) = p_0 + \epsilon \vec{1}_j$ (where $\vec{1}_j = (0, \ldots, 0, 1, 0, \ldots, 0) \in \mathbb{R}^J_+$, with 1 in the j-th place). Note that, as $\mathcal{P}$ is open and $p_0 \in \mathcal{P}$, it holds that $p(\epsilon) \in \mathcal{P}$ for $|\epsilon| > 0$ sufficiently small. Fix some $\mathbf{m}$. Using Assumption 1, without loss of generality, we can assume that $\Delta x^j_{i,i'}(\cdot) > 0$.

Define the function $\hat{\mathbf{m}}$ as

$$\hat{\mathbf{m}}(\epsilon) := e(\mathbf{m}, p_0, p(\epsilon))$$

Note that $\hat{\mathbf{m}}(0) = \mathbf{m}$, and that this function is continuous.

Let σ denote a permutation for which $\sigma^{-1} = \sigma$. Define the function $\tilde{\mathbf{m}}(\epsilon)$, whose components are given by

$$\tilde{m}_k(\epsilon) = e_{\sigma(k)}(e_k(m_k, p_0, p(\epsilon)), p(\epsilon), p_0) \tag{20}$$

Note that, by Pareto, for every ϵ, it holds that

$$(\mathbf{m}, p_0) \sim (e(\mathbf{m}, p_0, p(\epsilon)), p(\epsilon)) = (\hat{\mathbf{m}}(\epsilon), p(\epsilon))$$

By Income Anonymity

$$(\hat{\mathbf{m}}(\epsilon), p(\epsilon)) \sim (\sigma(\hat{\mathbf{m}}(\epsilon)), p(\epsilon)) \tag{21}$$

Using (21) and applying the Pareto indifference condition once again, we have that

$$(\hat{\mathbf{m}}(\epsilon), p(\epsilon)) \sim (\sigma(\hat{\mathbf{m}}(\epsilon)), p(\epsilon)) \sim (e(\sigma(\hat{\mathbf{m}}(\epsilon)), p(\epsilon), p_0), p_0)$$

Income Anonymity at p_0 then yields

$$(e(\sigma(\hat{\mathbf{m}}(\epsilon)), p(\epsilon), p_0), p_0) \sim (\sigma^{-1}(e(\sigma(\hat{\mathbf{m}}(\epsilon)), p(\epsilon), p_0)), p_0) = (\tilde{\mathbf{m}}(\epsilon), p_0)$$

By the transitivity of the indifference relation, it follows that

$$(\mathbf{m}, p_0) \sim (\tilde{\mathbf{m}}(\epsilon), p_0) \tag{22}$$

Note that $p(\cdot)$ is continuously differentiable, and that $\{e_k\}$ are continuously differentiable (by assumption). We therefore have that each $\tilde{m}_k$ is continuously differentiable at $\epsilon = 0$.

Thus, we can use the first-order Taylor expansion of $\tilde{m}_k$ at $\epsilon = 0$ to obtain

$$\tilde{m}_k(\epsilon) = \tilde{m}_k(0) + \epsilon \tilde{m}'_k(0) + o_k(\epsilon) \tag{23}$$

where $\lim_{\epsilon \to 0} o_k(\epsilon)/\epsilon = 0$.

Using (20), together with the identities $p(0) = p_0$ and $e_k(m, p_0, p_0) = m$, we can rewrite

$$\tilde{m}_k(0) = e_{\sigma(k)}(\underbrace{e_k(m_k, p_0, p(0))}_{m_k}, \underbrace{p(0)}_{p_0}, p_0) = m_k$$

The derivatives $\{\tilde{m}'_k(0)\}$ can then be written as

$$\tilde{m}'_k(0) = \frac{\partial e_{\sigma(k)}(m_k, p_0, p_0)}{\partial m} \frac{\partial e_k(m_k, p_0, p_0)}{\partial p_T^j} + \frac{\partial e_{\sigma(k)}(m_k, p_0, p_0)}{\partial p_S^j} \tag{24}$$

Note the following:

- As $e_k(m, p_0, p_0) = m$ for every k and m, it holds that

$$\frac{\partial v_k(m, p_0)}{\partial m} = \frac{\partial e_k(m, p_0, p_0)}{\partial m} = 1 \quad \forall k, m \tag{25}$$

- By Claim 3,

$$\frac{\partial e_k(m, p_0, p_0)}{\partial p_T^j} = x_k^j(m) \quad \forall k, m \tag{26}$$

- By Claim 3,

$$\frac{\partial v_k(m, p_0)}{\partial p^j} = \frac{\partial e_k(m, p_0, p_0)}{\partial p_S^j} = -x_k^j(m) \quad \forall k, m \tag{27}$$

Substituting (25), (26) and (27) into (24), we obtain

$$\tilde{m}'_k(0) = 1 \cdot x_k^j(m_k) - x_{\sigma(k)}^j(m_k) = \Delta x_{k,\sigma(k)}^j(m_k) \tag{28}$$

Substituting these expressions into (23), along with $\tilde{m}_k(0) = m_k$, we have

$$\tilde{m}_k(\epsilon) = m_k + \epsilon \Delta x_{k,\sigma(k)}^j(m_k) + o_k(\epsilon)$$

Substituting into (22), we obtain

$$(\mathbf{m}, p_0) \sim (\mathbf{m} + \epsilon(\Delta x_{1,\sigma(1)}^j(m_1), ..., \Delta x_{I,\sigma(I)}^j(m_I)) + o(\epsilon), p_0) \tag{29}$$

where $o(\epsilon) = (o_1(\epsilon), ..., o_I(\epsilon))$.

We now look at the particular case in which $\sigma = \sigma_{i,i'}$, that is, σ is the permutation that switches the incomes of individuals i and i', leaving all other incomes constant. In this case, we have that

$$\Delta x_{i,\sigma(i)}^j(m_i) = \Delta x_{i,i'}^j(m_i)$$
$$\Delta x_{i',\sigma(i')}^j(m_{i'}) = \Delta x_{i',i}^j(m_{i'}) = -\Delta x_{i,i'}^j(m_{i'})$$
$$\Delta x_{k,\sigma(k)}^j(m_k) = 0 \text{ for all } k \neq i, i'$$

We thus obtain

$$(\mathbf{m}, p_0) \sim (\mathbf{m} + \epsilon(0, ..., 0, \underbrace{\Delta x_{i,i'}^j(m_i)}_{\text{index } i}, 0, ..., 0, \underbrace{-\Delta x_{i,i'}^j(m_{i'})}_{\text{index } i'}, 0, ..., 0) + o(\epsilon), p_0) \tag{30}$$

It follows that, for $\epsilon \neq 0$ sufficiently small, the ordering is approximately indifferent with respect to simultaneously increasing the income of individual i by $\epsilon \Delta x^j_{i,i'}(m_i)$ and decreasing the income of individual i' by $\epsilon \Delta x^j_{i,i'}(m_{i'})$. It follows that the social marginal rate of substitution $MRS(m_i, m_{i'}|\mathbf{m}, p_0)$ is well-defined at $(\mathbf{m}, p_0)$ and given by the ratio $\Delta x^j_{i,i'}(m_{i'})/\Delta x^j_{i,i'}(m_i)$. Note that this result applies to all $\mathbf{m} \in \mathbb{R}^I_{++}$. That is, we have shown that, for each $\mathbf{m}$, this marginal rate of substitution is well defined at the point $(\mathbf{m}, p_0)$, and equal to $\Delta x^j_{i,i'}(m_{i'})/\Delta x^j_{i,i'}(m_i)$. Note that this marginal rate of substitution does not depend on the incomes of individuals other than i and i'.

Let $k \neq k'$ be two arbitrary individuals, and consider the social ranking of their incomes, m_k and $m_{k'}$, conditional on p_0 and $\mathbf{m}$. By Income Anonymity, it holds that

$$(\mathbf{m}, p_0) \sim (\sigma_{i,k}(\sigma_{i',k'}(\mathbf{m})), p_0)$$

It follows that the ranking of the incomes of individuals k and k' conditional on p_0 is the same as the ranking of the incomes of individuals i and i', conditional on p_0 (regardless of other individuals' incomes). Hence, the social marginal rates of substitution between the incomes of individuals k and k' (conditional on p_0) are given by $MRS(m_k, m_{k'}|\mathbf{m}, p_0) = \Delta x^j_{i,i'}(m_{k'})/\Delta x^j_{i,i'}(m_k)$, as in (11).

We have thus established that, if there exists a social ranking that satisfies our axioms then, for every $\mathbf{m}$ and every k, k', the marginal rates of substitution $MRS(m_k, m_{k'}|\mathbf{m}, p_0)$ are well defined at $(\mathbf{m}, p_0)$ and given by $\Delta x^j_{i,i'}(m_{k'})/\Delta x^j_{i,i'}(m_k)$. As marginal rates of substitution uniquely characterize a social ranking, it follows that there is only one ranking of income distributions *conditional on* p_0 that satisfies our axioms. Moreover, this conditional ranking is representable by the social welfare function

$$(m_1, ..., m_I) \mapsto \sum_{k=1}^{I} \int_1^{m_k} \frac{dm'}{|\Delta x^j_{i,i'}(m')|}$$

which is strictly increasing in each m_k and has social marginal rates of substitution that coincide with the social marginal rates of substitution that we characterized.

To conclude the proof of uniqueness, it is necessary to show that any two rankings that satisfy Pareto and agree on the ranking of budgets conditional on p_0 must also agree on the ranking of all other budgets (with prices that are different from p_0). But this follows trivially from the Pareto indifference condition, as it requires that, for every $(\mathbf{m}, p)$,

$$(\mathbf{m}, p) \sim (e(\mathbf{m}, p, p_0), p_0)$$

Hence, we have that

$$(\mathbf{m}, p) \preceq (\mathbf{m}', p') \Leftrightarrow (e(\mathbf{m}, p, p_0), p_0) \preceq (e(\mathbf{m}', p', p_0), p_0) \tag{31}$$

Thus, any two budget allocations must be ranked based on the ranking of their equivalent incomes at the reference prices. The uniqueness of the ranking conditional on p_0 therefore implies the uniqueness of the ranking of all budget allocations.

To conclude the proof, we need to show that the social ordering is represented by $W_{i,i',j}$. Using (3), note that the social marginal rates of substitution of $W_{i,i',j}(\cdot, p_0)$ are given by

$$MRS(m_k, m_{k'}|\mathbf{m}, p_0) = \frac{\frac{\partial W_{i,i',j}(\mathbf{m},p_0)}{\partial m_k}}{\frac{\partial W_{i,i',j}(\mathbf{m},p_0)}{\partial m_{k'}}} = \frac{\frac{1}{|\Delta x^j_{i,i'}(m_k)|}}{\frac{1}{|\Delta x^j_{i,i'}(m_{k'})|}} = \frac{|\Delta x^j_{i,i'}(m_{k'})|}{|\Delta x^j_{i,i'}(m_k)|} = \frac{\Delta x^j_{i,i'}(m_{k'})}{\Delta x^j_{i,i'}(m_k)}$$

(where the last equality follows from Assumption 1). Thus, the social marginal rates of substitution of the ordering represented by $W_{i,i',j}(\cdot, p_0)$ are the same as the marginal rates of substitution that we identified, conditional on p_0. As $W_{i,i',j}$ is a strictly increasing function of individuals' money-metric utilities, it satisfies the Pareto condition. Hence, this social welfare function represents the unique social ordering that satisfies our axioms.

C.1 The role of Assumption 1

Note that the proof relies only on the assumption that $|\Delta x^j_{i,i'}(\cdot)| > 0$, and not on the assumption that $|\Delta x^l_{k,k'}(\cdot)| > 0$ for all k, k', l. Thus, we have established that if $|\Delta x^j_{i,i'}(\cdot)| > 0$ for some i, i' and j, then there exists at most one social ordering that satisfies Pareto and Income Anonymity, and it is represented by (2). This applies also to preference profiles for which $|\Delta x^l_{k,k'}(m)| = 0$ for some $(k, k', l) \neq (i, i', j)$, including cases in which some individuals have exactly the same preferences.

The proof also demonstrates that if $\Delta x^j_{i,i'}(m_i) > 0$ and $\Delta x^j_{i,i'}(m_{i'}) < 0$ for some m_i and $m_{i'}$, then there cannot exist an ordering that satisfies the theorem axioms: by (30), such an ordering would have to have a *negative* marginal rate of substitution between m_i and $m_{i'}$, which would be a violation of the Pareto condition (because it should be desirable to increase the incomes of both individuals).

D Restricted budget domains

In the main text, individual budgets range over $\mathbb{R}_{++} \times \mathcal{P}$, and budget allocations range over $\mathbb{R}^I_{++} \times \mathcal{P}$. However, some common classes of preferences, including the Stone-Geary, Gorman

form and PIGL preferences in Section 4, are well-defined only for budgets with sufficiently high incomes. This appendix explains how our results extend to such restricted domains. Note also that $\mathcal{P}$ may be a small neighborhood around the reference prices, p_0, so that our results also apply as local characterizations.

Restricted domains. Let the set of individual budgets be an open set of the form

$$\mathcal{B} = \{(m, p) | p \in \mathcal{P}, m > \underline{m}(p)\}$$

where $\{\underline{m}(p) \geq 0\}_{p \in \mathcal{P}}$ are some constants, and let the set of budget allocations be

$$\mathcal{B}^* = \{((m_1, ..., m_I), p) | (m_i, p) \in \mathcal{B} \quad \forall i\}$$

Individual preferences are defined over $\mathcal{B}$, and Ω is the set of rankings of $\mathcal{B}^*$. In particular, all profiles in D are defined over the same set of budgets. We then restrict the statements of Assumption 1 and the axioms to budgets in $\mathcal{B}^*$.

Assumptions. We make two assumptions on the restricted domain. First, we assume that $\underline{m}(p) < 1$ for all $p \in \mathcal{P}$. This normalization guarantees that the lower limit of the integrals in (2) lies within the domain at p_0, and that budgets with an income of 1 are in $\mathcal{B}$, which is used in the proof of Theorem 2. When $\underline{m}(\cdot)$ is bounded on $\mathcal{P}$ (for example, when $\mathcal{P}$ is a bounded neighborhood of p_0), this assumption amounts to a normalization of the units of income. Second, we assume that equivalent incomes near the reference prices remain within the domain: for each $(m, p_S) \in \mathcal{B}$, there exists some $\epsilon > 0$ such that for all $p_T \in \mathcal{P}$ such that $||p_0 - p_T|| < \epsilon$, there exists some $e_i(m, p_S, p_T)$ such that $(e_i(m, p_S, p_T), p_T) \in \mathcal{B}$ and

$$(m, p_S) \sim_i (e_i(m, p_S, p_T), p_T)$$

In particular, $(v_i(m, p), p_0) = (e_i(m, p, p_0), p_0) \in \mathcal{B}$ for every $(m, p) \in \mathcal{B}$, so that the money-metric utility representations are well-defined.

Results. Under these assumptions, Claim 1 and Theorems 1 and 2 hold as stated, with $\mathbb{R}_{++} \times \mathcal{P}$ and $\mathbb{R}^I_{++} \times \mathcal{P}$ replaced by the restricted sets $\mathcal{B}$ and $\mathcal{B}^*$, and their proofs apply without change. Note that, as a ranking of a restricted domain compares fewer allocations, the uniqueness result in Theorem 1 is stronger on a restricted domain, while the existence result in Theorem 2 is weaker.

Examples. The restricted domains for the examples in Section 4 are as follows.

- **Stone-Geary preferences:** $\underline{m}(p) = \sum_j p^j \underline{x}^j$, that is, $\mathcal{B}$ is the set of budgets for which the subsistence bundle is affordable.
- **Gorman form preferences:** $\{\underline{m}(p)\}$ need to be high enough so that the functional form is valid, that is, nobody consumes a negative quantity of any good. In addition, we need to impose that $g(p)\underline{m}(p) + \gamma_i(p) \geq g(p_0)\underline{m}(p_0) + \gamma_i(p_0)$ for all i and $p \in \mathcal{P}$, which guarantees that equivalent incomes at p_0 are in $\mathcal{B}$.
- **PIGL preferences:** $\{\underline{m}(p)\}$ need to be high enough so that the representation is valid. In addition, we need to impose that $\frac{1}{\epsilon}\left(\frac{\underline{m}(p)}{h(p)}\right)^{\epsilon} - \gamma_i(p) \geq \frac{1}{\epsilon}\left(\frac{\underline{m}(p_0)}{h(p_0)}\right)^{\epsilon} - \gamma_i(p_0)$ for all i and $p \in \mathcal{P}$, which guarantees that equivalent incomes at p_0 are in $\mathcal{B}$.

E Proof of Theorem 2

First direction. We show that if there exists an ordering that satisfies Income Anonymity and Pareto, then the preference profile must satisfy the proportionality condition and the additive separability condition.

We start with the proportionality condition. If there exists an ordering that satisfies our axioms, then it must be represented by $W_{i,i',j}$ for every i, i', j. Note that each $W_{i,i',j}$ is additively separable in individuals' money metric utilities, $v_1(m_1, p), ..., v_I(m_I, p)$. Since additively separable representations are unique up to an affine transformation, it follows that there exist some $a_{i,i',j} > 0$ and $b_{i,i',j}$ such that

$$\int_1^{v_k(m,p)} \frac{dm'}{|\Delta x^j_{i,i'}(m')|} = \frac{1}{a_{i,i',j}} \int_1^{v_k(m,p)} \frac{dm'}{|\Delta x^1_{1,2}(m')|} + b_{i,i',j}$$

Taking the derivative with respect to $v_k(m,p)$ at $p = p_0$ (so that $v_k(m,p) = m$) implies

$$\frac{1}{|\Delta x^j_{i,i'}(m)|} = \frac{1}{a_{i,i',j}} \frac{1}{|\Delta x^1_{1,2}(m)|} \Rightarrow |\Delta x^j_{i,i'}(m)| = a_{i,i',j} |\Delta x^1_{1,2}(m)|$$

By Assumption 1, the proportionality condition follows (for $\alpha_{i,i',j} = a_{i,i',j}$ or $\alpha_{i,i',j} = -a_{i,i',j}$).

We now show that if there exists an ordering that satisfies Income Anonymity and Pareto, then the additive separability condition must hold. Theorem 1 guarantees that if such an ordering exists then, for every $i \neq 1$, it is represented by

$$W_{1,i,1}(\mathbf{m}, p) = \sum_{k=1}^{I} \int_1^{v_k(m_k,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|}$$

For this ordering to satisfy Income Anonymity, the value of $W_{1,i,1}$ should be the same if we were to switch the incomes of individuals 1 and i. If we fix the income of individual 1 to $m_1 = 1$ and let $m_i = m$, this implies that, for every m and every p,

$$\int_1^{v_1(1,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|} + \int_1^{v_i(m,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|} = \int_1^{v_1(m,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|} + \int_1^{v_i(1,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|} \Rightarrow$$
$$\int_1^{v_i(m,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|} = \underbrace{\int_1^{v_1(m,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|}}_{f_i(v_1(m,p))} + \underbrace{\int_{v_1(1,p)}^{v_i(1,p)} \frac{dm'}{|\Delta x^1_{1,i}(m')|}}_{g_i(p)}$$

Note that the left hand side is a representation of individual i's preferences (because this is a strictly increasing transformation of $v_i(m,p)$). It follows that these preferences have a continuously differentiable representation that is additively separable in $v_1(m,p)$ and p (note that f_i and g_i are continuously differentiable, as required).

Second direction. We now show that if the two conditions hold, then there exists an ordering that satisfies our axioms. Fix some $i \neq 1$ and let $f_i(v_1(m,p)) + g_i(p)$ denote the additively-separable representation of individual i's preferences (as guaranteed by the additive separability condition, with f_i and g_i continuously differentiable).

Note that, as $v_i(m,p)$ is defined as the money-metric utility of individual i at prices p_0, individual i must be indifferent between $(v_i(m,p), p_0)$ and (m,p):

$$f_i(v_1(m,p)) + g_i(p) = f_i(v_1(v_i(m,p),p_0)) + g_i(p_0)$$

Note that $v_1(m',p_0)$ is individual 1's money-metric utility at the reference prices, p_0. Hence, $v_1(m',p_0) = m'$ for all m'. It follows that $f_i(v_1(v_i(m,p),p_0)) = f_i(v_i(m,p))$ and hence

$$f_i(v_1(m,p)) + g_i(p) = f_i(v_i(m,p)) + g_i(p_0)$$

Taking the derivatives of both sides with respect to p^j at $p = p_0$, we obtain

$$f_i'(m)\frac{\partial v_1(m,p_0)}{\partial p^j} + \frac{\partial g_i(p_0)}{\partial p^j} = f_i'(m)\frac{\partial v_i(m,p_0)}{\partial p^j}$$

$$\Rightarrow \frac{\partial g_i(p_0)}{\partial p^j} = f_i'(m)\left(\frac{\partial v_i(m,p_0)}{\partial p^j} - \frac{\partial v_1(m,p_0)}{\partial p^j}\right) = f_i'(m)\Delta x^j_{1,i}(m) \tag{32}$$

(where the last equality follows from (27)).

Thus, denoting $a = |\frac{\partial g_i(p_0)}{\partial p^j}|$ and noting that $a > 0$ and $f_i' > 0$,[21] it follows that

$$f_i'(m) = \frac{a}{|\Delta x_{1,i}^j(m)|} \Rightarrow$$
$$f_i(m) = \int_1^m \frac{a}{|\Delta x_{1,i}^j(m')|} dm' + b = \int_1^m \frac{a}{|\alpha_{1,i,j}\Delta x_{1,2}^1(m')|} dm' + b$$

where the last step follows from the proportionality condition.

Hence, applying a strictly increasing transformation to the representation we started with, we have that individual i's preferences are represented by

$$\frac{|\alpha_{1,i,j}|}{a}\left(f_i(v_1(m,p)) + g_i(p) - b\right) = \underbrace{\int_1^{v_1(m,p)} \frac{dm'}{|\Delta x_{1,2}^1(m')|}}_{f(v_1(m,p))} + \tilde{g}_i(p) \tag{33}$$

where $\tilde{g}_i(p) = g_i(p)(|\alpha_{1,i,j}|/a)$.

Consider the social welfare function

$$W_{1,2,1}(\mathbf{m}, p) = \sum_{i=1}^{I}\left(\int_1^{v_1(m_i,p)} \frac{dm'}{|\Delta x_{1,2}^1(m')|} + \tilde{g}_i(p)\right) = \sum_{i=1}^{I} f(v_1(m_i,p)) + \sum_{i=1}^{I}\tilde{g}_i(p)$$

This social welfare function represents a Paretian ordering because it is a sum of individual utility functions, in the representations that we established above. It is also symmetric in $m_1, ..., m_I$, and hence it satisfies Income Anonymity. It follows that such an ordering exists, as required by our theorem.

E.1 A violation of the additive separability condition

This section lays out an example of a preference profile that does not satisfy the additive separability condition. The purpose of this is to show that this condition can be violated.

Consider an example in which $v_1(m,p) = m/p^1$ and $v_2(m,p) = (m - p^2)/p^3$ (that is, individual 1 consumes only good 1, and individual 2 has Stone-Geary preferences over goods 2 and 3).[22]

Assume by way of contradiction that the additive separability condition is satisfied. Then, there exist continuously differentiable functions, f and g, such that $f(m/p^1) + g(p)$ is a

[21] To see this, note that f_i must be strictly increasing because $f_i(v_1(m,p_0)) = f_i(m)$, and individual i's preferences are increasing in income. Therefore $f_i' \geq 0$. If it were the case that, for some m, $f_i'(m) = 0$, we would obtain from (32) that $a = \frac{\partial g_i(p_0)}{\partial p^j} = 0$ and hence, from (32), we would have to conclude that $f_i'(m) = 0$ for *every* m, which would then imply that f_i is constant in m — contrary to the fact that f_i is strictly increasing. It therefore follows that $f_i' > 0$ and $a > 0$.

[22] The domain of budgets is restricted, as in Appendix D, so that $(m - p^2)/p^3 > 0$.

representation of individual 2's preferences. As individual 2 is indifferent with respect to changes in the price of good 1, it follows that

$$0 = \frac{\partial}{\partial p^1}(f(m/p^1) + g(p)) = -f'(\frac{m}{p^1})\frac{m}{(p^1)^2} + \frac{\partial g(p)}{\partial p^1}$$

$$\Rightarrow f'(\frac{m}{p^1})\frac{m}{(p^1)^2} = \frac{\partial g(p)}{\partial p^1}$$

$$\Rightarrow f'(\frac{m}{p^1}) = \frac{(p^1)^2\frac{\partial g(p)}{\partial p^1}}{m} \tag{34}$$

In addition, note that individual 2's marginal rate of substitution between m and p^2 is given by

$$\frac{\frac{\partial((m-p^2)/p^3)}{\partial m}}{\frac{\partial((m-p^2)/p^3)}{\partial p^2}} = \frac{1/p^3}{-1/p^3} = -1$$

Thus, we must have that, for every m,

$$\frac{\frac{\partial\left(f(\frac{m}{p^1})+g(p)\right)}{\partial m}}{\frac{\partial\left(f(\frac{m}{p^1})+g(p)\right)}{\partial p^2}} = -1 \tag{35}$$

$$\Rightarrow \frac{\partial\left(f(\frac{m}{p^1}) + g(p)\right)}{\partial m} = -\frac{\partial\left(f(\frac{m}{p^1}) + g(p)\right)}{\partial p^2} \Rightarrow$$

$$f'(\frac{m}{p^1})\frac{1}{p^1} = -\frac{\partial g(p)}{\partial p^2} \Rightarrow f'(\frac{m}{p^1}) = -\frac{\partial g(p)}{\partial p^2}p^1$$

Using (34), it follows that

$$-\frac{\partial g(p)}{\partial p^2}p^1 = \frac{(p^1)^2\frac{\partial g(p)}{\partial p^1}}{m}$$

Note that the left hand side does not depend on m; hence, the right hand side cannot depend on m, and hence we must have

$$(p^1)^2\frac{\partial g(p)}{\partial p^1} = 0$$

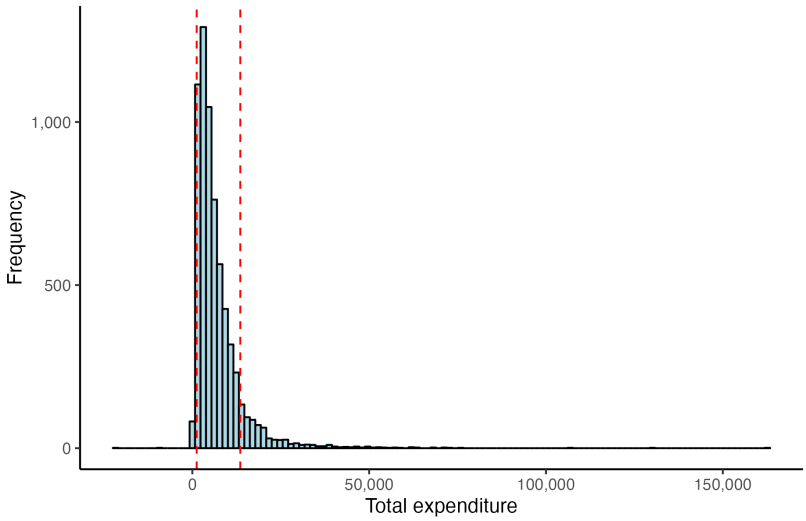
which, by (34), implies that $f'(\cdot) \equiv 0$. Hence, $f(m/p^1)$ is some constant that does not depend on m/p^1. But this is a contradiction, as individual 2's preferences are strictly increasing in income.

F Empirical estimates

Table 1: Descriptive statistics for expenditure categories in the final sample

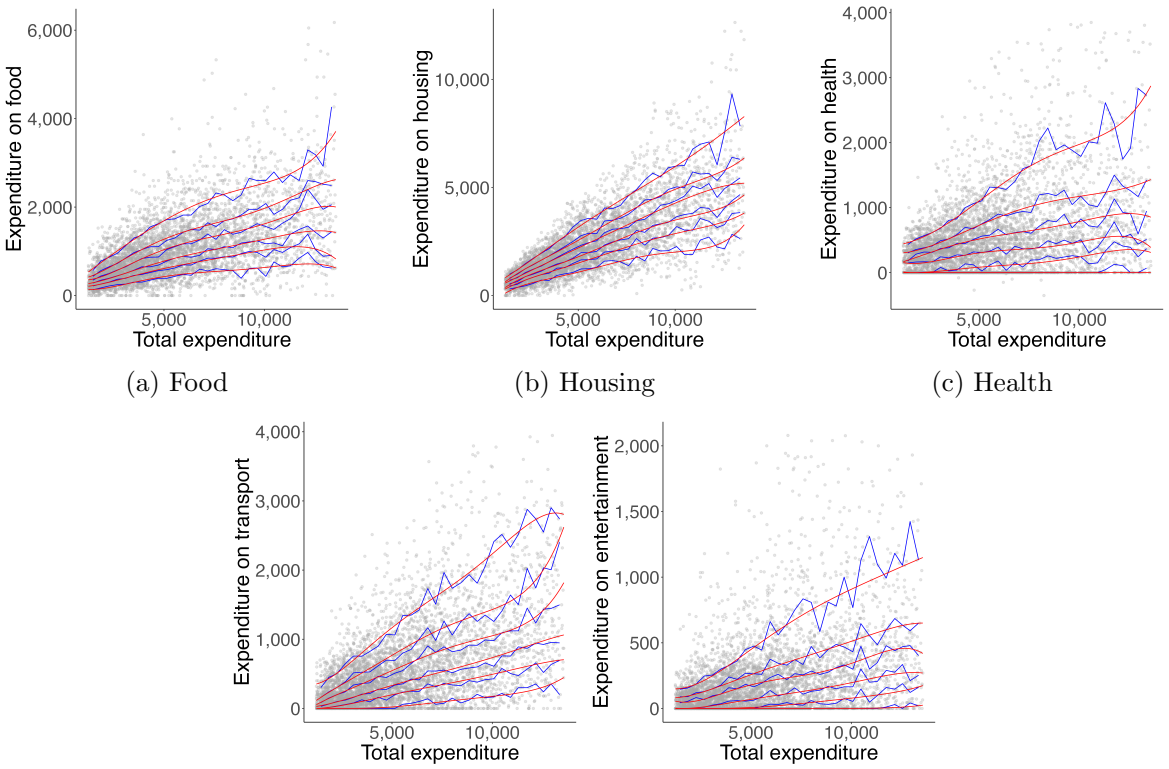
Expenditure category	Mean	Median	Min	Max
Housing	2,240	1,840	0	12,638
Food	968	780	0	6,175
Transport	577	378	0	10,085
Health	524	352	−353	8,988
Insurance and pensions	490	153	0	9,697
Entertainment	211	105	0	5,282
Cash contributions	158	0	0	7,910
Alcohol	57	0	0	3,500
Apparel	54	0	0	3,000
Tobacco	43	0	0	3,900
Personal care	36	0	0	1,175
Education	16	0	0	7,778
Reading	9	0	0	900
Misc.	56	0	0	8,000

Figure 5: Distribution of total expenditures before removing extreme values



Note: Observations are restricted to single consumers. Dashed lines represent expenditure cutoffs in our data.

Figure 6: Percentiles of the distributions of expenditures on different categories as a function of income



(a) Food (b) Housing (c) Health

(d) Transportation (e) Entertainment

Note: Gray dots represent observations in our dataset. Red lines represent the estimated category-specific expenditure as a polynomial function of total expenditure for the 90th, 75th, 60th, 40th, 25th, and 10th percentiles. Red lines correspond to 5th order polynomial approximations, and are estimated through quantile regressions. Blue lines represent the empirically observed expenditure quantiles for each of 30 total expenditure groups, for the percentiles indicated above. These empirical quantiles are constructed by dividing the sample into 30 total expenditure bins, calculating the sample quantile within each bin (via a constant-only quantile regression), and connecting these within-bin estimates across the distribution with line segments. For ease of visualization, observations in the highest .5% expenditure levels in transport, health, and entertainment were dropped from the scatterplot.