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Marc Flandreau
University of Pennsylvania
mfl@sas.upenn.edu

Geoffroy Legentilhomme
Universiy of Zurich
geoffroy.legentilhomme@hist.uzh.ch

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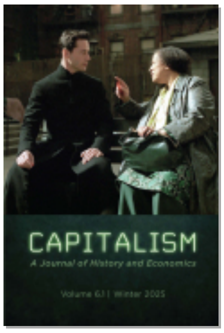

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Capitalism as Calculus: An Archaeology

Marc Flandreau
University of Pennsylvania

Geoffroy Legentilhomme
University of Zurich

> Compu'ter. n.s. [from compute.] Reckoner; accountant; calculator.
> "I have known some such ill computers, as to imagine the
> many millions in stocks so much real wealth." Swift.
>
> —*Samuel Johnson,* Dictionary of the English Language, *1755*

One puzzle in the history of modern economics concerns its mathematization. In an influential book, Philip Mirowski traced the origin of this phenomenon to neoclassical economists, who were the first to articulate a comprehensive project aimed at formalizing economics. In proceeding in this fashion, they would have been victims of a case of "physics envy." Emulating physics was their aim, and so they tried to develop a form of social

Note: This paper developed over the course of several years initially under the title "Governing the Computers: Towards an Economic History of the History of Economics?" We benefited from presentations of earlier drafts in conferences in Rome (Academia Nazionale dei Lincei, Palazzo Corsini, 5th Thomas Guggenheim Conference in the History of Economic Thought), Sciences Po Lille (Flandreau's keynote address at the 23rd ESHET Conference, 2019), and the Graduate Institute for International Studies and Development in Geneva (Pierre Dubois Conference on the History of Sovereign Debt, 2024), as well as seminars at Berkeley (2019), Yale (2019), and Rutgers (2021). Some of the ideas from the original draft were published previously ("Cyberpunk Victoria," *Economic History Review*, in 2022). Many thanks to our colleagues who provided enthusiasm and criticism, especially Carolyn Biltoft, Mike Bordo, Brad DeLong, Rui Esteves, Will Goetzmann, Tim Guinnane, Donald MacKenzie, Cristina Marcuzzo, Jeanette Rutterford, Margaret Schabas, and Eugene White. At the Institute and Faculty of Actuaries, London, we benefited from the support of Kenneth Bogle and David Raymont, librarians. We gratefully acknowledge generous financial support from the Howard S. Marks Chair of Economic History (University of Pennsylvania) and from the University Research Priority Program (URPP) Equality of Opportunity (University of Zurich).

physics. Using energy as metaphor, they came up with the concept of utility and, with this, they naturalized the use of mathematical symbols.[1]

While this may be so, we note that in the seventeenth and eighteenth centuries, mathematics was already at the center of economic life. Formulae and algorithms aimed at pricing assets were appearing in treatises on algebra and providing entries in almanacs. The same formulae informed the construction of practical methods, embedded in tables, that helped traders figure out values, sometimes in very detailed ways. More generally, a culture of computing was taking solid root in European capitalism. This emerging mathematics of wealth was pragmatic. It was a knowledge that did things: People were doing things with equations. In fact, computing participated in the rise of what Fernand Braudel called the "material civilization" of capitalism.

Using a terminology popularized by anthropologists Kenneth L. Pike and Marvin Harris (who borrowed it from linguists), the two "projects" alluded to above may be cast as being respectively "etic" and "emic."[2] "Etic" refers to an approach whereby the researcher defines the terms in which the problem is conceived. The economist's formalism offers an example: Since its beginnings, it has been meant to comprehend the "objective" facts of the economy. As economists have often emphasized, the project does not need to posit that individuals thought with equations: It's enough that everything happened as if they did. "Emic" by contrast refers to an approach where the problem is conceived in the actors' terms. It suggests that these terms are important to understand reality. From this vantage point, it is important to recognize that, historically, equations began to govern the minds of economic agents long before neoclassical economics was invented. Study of the grassroots expansion of mathematics and calculus as tools to assist transactions is an emic project, and this is the one we pursue here.

This article busies itself with financial calculus as it was performed from the onset of the Financial Revolution (the late seventeenth century) to the apex of the Victorian era (the late nineteenth century). The geographical focus is England, the beating heart of Western capitalism. We place ourselves at the level of the individual "computer" defined in the opening quote (Dr. Johnson's *Dictionary* also gave the term "computist," which fell into oblivion). In the story we tell, human computers provided the elementary parts of a capitalist machinery geared toward computation as deal-making. They performed the

[1] Mirowski, *More Heat than Light*; Schabas, "What's So Wrong with Physics Envy?" for a critical review.

[2] Pike, *Language*; Harris, "Emic/Etic Distinction."

work of value, a price discovery technique. Users wanted to be shepherded by routines. They relied on mathematicians, algorithms and tables to show them the way forward. Cast as work of value, capitalism becomes the set of technologies steering users away from the pitfalls of ill calculus.

While pragmatic, the computing knowledge that paved the way for the growth of capitalism was sophisticated in its design and lofty in its ambitions. But unlike what would happen with the mathematics of neoclassical economists, whose lofty ambitions led them to preoccupy themselves with the production of a theory that would lead to predictions relative to "objective" prices, the early mathematics of wealth wanted to help parties strike a bargain. When contemporaries described the prices that could be derived from their mathematical formulae, they described them (positively) as being "agreeable." The judgment was both aesthetic (as in "pleasant") and commercial (as in "the price at which we will agree"). To that end, computers stood watch on value-making and, by facilitating adjudication, helped reach this apex of financial commerce—the transaction.

We argue that, historically, computation has been a means of "making" capitalism. It was in fact a form of brokerage that helped to make markets. Accordingly, we posit that computers were *financial intermediaries* in the literal sense of the term—they were agents tasked with resolving information asymmetries. This is what made mathematics so essential to capitalist development. Automation reduced the scope for manipulation, enabling traders to see eye to eye as it lit up a pathway to robust exchange. The reason why numerical technologies were attractive, in other words, was not that they conferred a precision that would have better empowered end users to make more informed investment decisions. The number of decimals which started being demanded very early on, when that precision seems ludicrous, was a performance of the trustworthiness of the intermediary.

Because "trust" is such an overused term, let's try and be precise: The kind of trust which the computing machinery commanded was akin to the trust one has in the justice system. The connotation here is with medieval theories of the just price, explaining why, in addition to calling machine-produced prices "agreeable," contemporaries often spoke of their "equity"—they wanted such prices to be "equitable." Mathematization was desirable as a form of adjudication. Computing did not resolve a price difference but a price *different*. It reassured users that, by surrendering the asset at the price determined by the numerical oracle, they would not be taken advantage of. The exact price here was much less important than the belief that an exact price was possible and the computing equipment one relied upon.

We argue that retracing the history of how computers adjudicated value does much more than contributing an interesting but arcane angle to the history of capitalism. By enabling a process of transmutation of things into values and vice versa, computing *simultaneously* shaped how traders interacted. It shaped how markets were made, and (most importantly perhaps) it shaped what was traded. The point is that long before Nvidia and DeepSeek entered our vocabulary, computing began organizing society, in ways not always visible to the naked eye of the passing observer. Against this backdrop, the often-noted fact that since the early modern age, throughout the Victorian era, and all the way to the twentieth century, the human calculator has been called a "computer" is not merely an amusing coincidence. It is the whole story: a proof that capitalism is a computing relation—and thus itself a computer—and should be studied as such.

The simultaneous histories of scientific knowledge and the capital market proposed here is at heart a theory of transaction costs. The precise principle we will exploit to guide ourselves states that there exists a deep correspondence between the *instruments* traded in the capital market and the *methods* developed to adjudicate their value. The reason for the existence of this two-way relation is the following: On the one hand, a condition for a given financial instrument to prosper is a dedicated numerical ecology. On the other, this ecology expands as the demand for price adjudication of the financial instrument in question increases. Game theorists describe this as strategic complementarity. This may be called the "lamppost theorem." Given the fact that there is one lamppost in the street, the safest place to lose one's keys is just under the lamppost.

Strategic complementarity operates as a selection principle: In the context we consider, it states that what exists, what we observe, is computable. Perhaps the financial product came first or perhaps the computing method came first. But this does not really matter because they are joined at the hip: The instrument and the numerical formula form a pair. From the onset, financial instruments were conceived to surrender amicably to routines that facilitated their agreeable pricing. It would be flawed, therefore, to imagine financial products as natural objects emerging independently, and as the objects of a "natural" science of finance. The computer is the backstory of capitalism, not its divine surprise.

This provides the elements of a phenomenology. Strategic complementarity predicts dominance, which encourages us to investigate the jumble of financial products to find a dominant template. As it turns out, it consisted in something known in the early modern world as the "annuity." Harkening back to the Commercial Revolution at least, the annuity was a *contract* to pay a fixed amount of money for a given number of years. The annuity was both an object and a *problem*, the latter formalized in 1202 by Italian mathematician Leonardo Fibonacci, who showed that the contract could be represented by an equation whose unknown was the yield (or return, or interest rate). Fibonacci's equation defined implicitly the yield as a function of the value. Actual calculation was difficult, inviting the intervention of a skilled computer, and *this* was the revolution.

In this fashion, the annuity (and its mathematics) was placed at the center of the trade in financial pledges, with remarkable results. Over the following centuries and accelerating during the Financial Revolution, considerable efforts would be expended to enhance the speed and accuracy with which computers would resolve the annuity problem, ultimately reaching considerable efficacy during the Victorian era. Conversely, enhanced computability secured a special role for the annuity as the central code of capitalism. The annuity would keep reproducing itself through the agency of avatars that would "mimic" the structure of the annuity to take advantage of its preexisting numerical infrastructure. These avatars were "derivatives" in the modern sense of the term—financial contracts whose value was dependent on an underlying asset, the annuity.

At a broad level this explains why "fixed-income securities," as we would today call the annuity family, dominated the financial landscape at the inception of modern capitalism. The early prevalence of fixed income instruments has been noted previously but not elucidated. Our emphasis on computing suggests that what was so attractive with such instruments was that trading in them was accessible to anyone, or at least to anyone capable of employing a computer. The mathematics of the annuity shed light on numerical opacities that would otherwise have stood in the way of traders attempting to swap instruments with different payment profiles. Capitalism began with the mathematics of time and the mastery of those two cardinal operations, actualizing and compounding, which set the terms of exchange between present and future. A deep market for stocks, with its hard-to-comprehend dividends and endless list of information asymmetries which mathematics alone could not resolve, would be much slower to come.

At a practical level, the dominance of the annuity provides an Ariadne's thread to the study of the numerical underpinnings of capitalism. Its centrality means that there is abundant material to document the accumulation of computing power. As suggested above, computable capitalism has left plenty of traces in the shape of a rich literature providing ample material on the range of computing "technologies" extant. Turning to this material renders it possible to measure long-term productivity gains achieved along the accuracy frontier. What is more, unpacking the economics of the evolving numerical food chain illuminates a companion development of the progress of formulae, consisting in constant efforts at improving the *computers* themselves. Computers could not just be left alone to run the programs according to their whims. They had to be *governed*. Matching efforts to improve the programs, the computers were selected, trained, updated, etc.

Three computing orders succeeded one another between the onset of the Financial Revolution in the late seventeenth century and the Victorian era. The first was steered by the mathematicians of wealth; the second was dominated by stockjobbers, the market makers who controlled the London Stock Exchange; the third was operated by actuaries, who professionalized the work of value. These computing orders may be understood, equivalently, as ways to govern markets, ways to govern value, and ways to govern calculators. Recalling the central object of mathematization—to ensure the manufacturing of agreeable prices—the primordial objective of these successive regimes was to manage the public image of computing. This included handling of price scandals, fallout from rogue computers, and mitigating the risks posed by hackers. Countermeasures were developed to handle such situations, ultimately shaping the supply of computing.

Crucially, each computing order must be understood in a systemic fashion: not as a stage of development of science, but as a stage of development of the capital market. The distribution of computing services was determined by the structure of the capital market. Over time, as the capital market got increasingly centralized—here the rise of the London Stock Exchange was the key development, with tremendous consequences for the destiny of financial calculus, as we shall see—price discrepancies became more visible and less tolerable. Responding to this situation was the onset of a process of formatting, the industrialization of analogue computers. As the work of trading demanded ever-rising levels of precision, computers were subjected to increasing travails.

So, what does all this allow us to see that we have not so far seen in the history of financial capitalism? It is that conventional descriptions of finance as dematerialization are misleading. It is not just that capitalism has a material infrastructure. Plainly, it's that capitalist accumulation is a process of *materialization*. As computation grasped value, it grasped the traders, the sites of trading, the stuff traded, and the valuers. Capitalism produced, for instance, a *materialization* of time, observed in the emergence of ur-physical securities that procured, in fact, exposure to infinity. Retracing the coevolution of the numerical techniques that enabled sorting of value, and of the markets in which these values were swapped for one another, provides us with a front-row seat where we can observe computing and, beyond this, the way capitalism, a material object, was coded.

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It may be said that this study returns to very old questions. In a passage from the second volume of *Material Civilization*, Fernand Braudel remarks that economic historians shall "underestimate" at their own peril the amount of computing knowledge which capitalist accumulation requires."[3] A few years before, P. G. M. Dickson had provided a premonition, underscoring the import of early modern actuarial knowledge in laying the foundation of his *Financial Revolution*. Dickson devoted a full chapter to contemporary understandings of the interest rate.[4] In fact, at the time both Braudel and Dickson were writing, there seems to have existed a widespread understanding among cultural historians of the relevance of computing, as illustrated for instance by Natalie Zemon Davis's excellent study of French commercial arithmeticians.[5]

Considering that coding is a language, it is strange that the linguistic turn estranged research from this early awareness. Affirming a general disdain of economic analysis, subsequent historians have systematically sought to distance themselves from economic factors and in particular transaction costs. Consider as illustration a classic of the modern genre, Steven Shapin's *Social History of Truth*. It emphasizes a *cultural* conception of truthfulness. As Shapin explains, gentlemen scientists (and as a result "Gentlemanly Science") were born out of the aura their social position commanded. This hardly disproves the import of underlying economic factors. The mechanics

[3] Braudel, *Civilisation matérielle*, 360. Also, in calculus, abaci and Fibonacci, pp. 509 ff. Sanford, *Short History of Mathematics*, 116–41, discusses the relation between mathematics and early modern finance.

[4] Dickson, speaks of the "developing study of actuarial science, generating greater confidence of the private insurance, municipal finance and long-term government borrowing" (*Financial Revolution*, 46). See also his discussion of Archibald Hutcheson (101–2) and chap. 19 ("The Rate of Interest in Theory and Practice") on contemporary understandings of fixed-income securities.

[5] Zemon Davis, "Sixteenth-Century French Arithmetics"; Sanford, *Short History of Mathematics*, provides evidence of the awareness of the relation between mathematics and early modern finance.

of trustworthiness alluded to by Shapin can be recast in economic language. Belief in one another's word is established based on the estimates that are formed of the person upon whom one depends for information. Status was used to signal trustworthiness and in this manner facilitate the distribution of truth. This undercover economic theory of information is perfectly amenable to economic analysis and, in fact, not inconsistent in its general logic with the theory we articulate here: The gentleman scientist is a broker of sorts.[6]

One could very well follow down the path of this intuition and construct a theory of knowledge as market intermediation. This approach however was explicitly resisted by previous generations of historians of science, even as their language abounds with economic terms—not only "trust," but "beliefs," "strategies," "moral economies," etc.[7] Take the work of Lorraine Daston who made extensive contributions to the history of insurance. She sees a disconnection between the life insurance industry and its mathematics, the former having developed "empirically" and (Daston adds) without any guidance from the mathematicians. According to Daston, "the influence of [the] mathematical literature on the voluminous trade in annuities and insurance was negligible until the end of the eighteenth century."[8]

But how can we test whether the "mathematical literature" influenced the trade in "annuities and assurance," or not? Daston seems to have taken a relative absence of *direct* evidence as evidence of absence. But there does exist ubiquitous early *indirect* traces that financial mathematics were embedded in the manner contemporaries approached valuations. In 1720, when Archibald Hutcheson made the case that the prices of the South Sea Company were inflated, he directed his readers to an annuity table where they would discover for themselves the kind of absurd expectations regarding dividends current prices implied. Assuming readers fully conversant with financial calculus, Hutcheson used the latter as a technique of persuasion.[9]

[6] Flandreau *Anthropologists*, 127–42.

[7] This program is present in a vast literature. We mention especially for their relevance to the present paper Ashworth, "Calculating Eye"; Daston, "Moral Economy of Science"; Daston and Galison, *Objectivity*; Schaffer, "Astronomers Mark Time."

[8] Daston, "Domestication of Risk"; Daston, *Classical Probability*. Compare Dickson, who states that "the theoretical work needed to lay the foundations of modern actuarial science (and hence life assurance) was begun by Pascal, Fermat, Huygens, de Wit, and especially Halley, van der Burch, de Moivre, and Simpson" (*Financial Revolution*, 41).

[9] Hutcheson, *Calculations Relating to the Proposals*, 2–6.

Likewise, M.E. Ogborn explains that the habit of life insurance companies to advertise their employment of a house actuary was a reaction to the distrust arising from the insurance bubble of the 1760s and 1770s. In both cases an episode of speculation invited a re-anchoring of beliefs, indirectly revealing the prestige mathematics had had all along and its role in stabilizing values in a crisis.[10]

The financial crisis of 2008, by giving birth to a so-called New History of Capitalism, has led to a healthy return of interest in the relations between capitalism and its forms of knowledge. This has resulted in some inspiring works focused on numerical culture at the onset of modern development, as well as contributions devoted to the "work of science."[11] Jessica Otis's book *By the Numbers* is a fascinating study of numeracy in early modern England, underscoring its import for business, material culture, and other aspects of social life. Otis offers valuable insights into the early modern proliferation of arithmetic treatises concerned with financial issues, such as interest rate calculation.[12] We note also the series of articles by William Deringer devoted to the numerical culture of early modern England. He provides fine-grained studies on a variety of relevant subjects, including mercantile epistemologies, early modern discounting technologies, and, more recently, the use of interest rate tables.[13]

In recent years, actuaries-turned-historians have provided particularly significant light. Craig Turnbull's history of actuarial thought is a valuable reference.[14] An especially important contribution is David Bellhouse's works on Abraham de Moivre, his circle, and the contribution that early modern mathematicians made to the development of life insurance.[15] Challenging Daston's contention that the progress of insurance had little to do with contemporary advances in financial mathematics, Bellhouse demonstrates the prior role played by the market for real assets (land leases).[16] Because the pricing of life insurance contracts posed problems formally identical to

[10] Ogborn, "Professional Name," 235–36. As Ogborn further explains, calculator James Dodson had been involved since the conception of the company in 1756.

[11] Rieppel, Lean, and Deringer, "Introduction."

[12] Otis, *By the Numbers*, 71, 78.

[13] Deringer, "Pricing the Future"; Deringer, "It Was Their Business to Know"; Deringer, "Mr. Aecroid's Tables."

[14] Turnbull, *History of British Actuarial Thought*.

[15] Bellhouse, *De Moivre*; Bellhouse, *Leases for Lives*; Bellhouse et al., "De Moivre's Knowledge Community."

[16] Bellhouse, *Leases for Lives*, 1–7.

those raised by "older" land leases, the mathematical solutions developed to deal with the latter were applied to sort the former. As we shall argue, this is a characteristic instance of knowledge transplant mobilizing strategic complementarities.[17]

Another historian whose research we ought to mention is Theodore M. Porter. Porter made famous contributions to explaining the secular rise of quantification. Like what we do here, Porter interprets quantification as a strategy for overcoming distrust.[18] His article "Quantification and the Accounting Ideal in Science" shows that, in the case of accounting, quantification emerges as an *objective* mechanism to legitimize the financial system in a time of distrust in its functioning. Reliance on quantification is stabilizing, he contends, because it embeds a mechanism constraining information production.[19] The great traction of numbers is that their production ensconces agents in an algorithm—one would speak today of a "smart contract"—limiting discretion and, by the same token, taking care of concerns over manipulation.[20]

Sociologists—especially economic sociologists and sociologists of science—discuss questions that are related to where this paper intends to go. For instance, Callon's work emphasizes the creative powers of norms, a process described as "performative."[21] The notion that mathematical formulae fulfill a market-making function is central to Donald MacKenzie's classic study of the Black-Scholes formula. It argues that the formula was not meant to *describe* how options were priced but to *enable* the new market for derivative trading.[22] This later intuition distinctly resonates with what we argue here, with the not small nuance that we provide an economic theory—rather than a sociological one—of the origins of the market-making role of mathematics. Crucially, we situate this development far back in time and we argue that blue-blooded mathematicians, rather than

[17] Bellhouse, *Leases for Lives*, in particular chapters 6, 7, 8, and 9. Bellhouse discusses aspects of the "consulting" work of mathematicians. The mathematicians of wealth also developed a clientele of wealthy people who bankrolled their research as they assisted these clients with pricing expertise. Bellhouse shows that the prestige of the mathematicians of wealth was mobilized in the 1760s and 1770s, when anxieties emerged regarding the pricing of life annuities.

[18] See Porter, *Trust in Numbers*, esp. chap. 5, "Experts Against Objectivity: Accountants and Actuaries," which deals with British actuaries. See also Alborn, "Calculating Profession."

[19] See also Porter, "Quantification and the Accounting Ideal."

[20] Similar ideas may be found in Allen, *Institutional Revolution*.

[21] Callon, "Introduction"; Latour emphasizes the role of "centers of calculation" (*Science in Action*, 233).

[22] MacKenzie and Millo, "Constructing a Market"; MacKenzie, *An Engine, Not a Camera*.

economists posturing as scientists, shepherded an ancient "grassroots" demand for formalism.[23]

Donald MacKenzie has recently described the competition of supercomputers, which play the role of market makers in the modern financial system, proposing to speak of the "material political economies of finance."[24] John Handel, in a paper that foregrounds the themes of a forthcoming book advocates historical study of the materiality of market structures and of their "politics."[25] This paper endorses these efforts and proposes to escalate the conversation: The point here is not only to acknowledge the import of materiality. What we need is a general theory of how the code of capital produced its own material universe, including, as we shall see, by programming the human element. For the historian of capitalism, we suggest, the material political economy of finance is just as well an anthropology.

In the body of the paper, we will engage the work of economic historian Joel Mokyr. In a series of popular books, Mokyr argues that cultural attitudes emerging in the late seventeenth and eighteenth centuries, favoring open scientific exchange and the accumulation of useful knowledge, prompted the growth of a knowledge-based economy. One chapter from his *Enlightened Economy* in particular offers finance as a case study. Mokyr ascribes advances in this service industry to developments in algebra and probability, which (to use Mokyr's expression) placed "commercial computation on a scientific basis."[26] This is a strange way to cast the matter because, as we will show, mathematics were at the root of the construction of the financial instruments. In fact, the instruments were designed so as to be resolved by mathematical methods extant. Computation did not need to be put on a scientific basis—it had developed on mathematical knowledge. As for the "new" methods developed in the nineteenth century, they were nothing but an improvement in the power of available techniques to resolve the same old problems.

Finally, we mention briefly the work of economists and historians of economic thought, to which we will return to it as needed subsequently. Useful

[23] Compare MacKenzie, Muniesa, and Siu, *Do Economists Make Markets?*

[24] MacKenzie, "'Making,' 'Taking'"; MacKenzie, *Trading at the Speed of Light*.

[25] Handel, "Material Politics of Finance." See also his still embargoed dissertation, Handel, "Infrastructures of Finance." The call, while welcome, is not new in economic history, where there does exist a whole literature dedicated to the microstructure of financial markets.

[26] See also Mokyr, who further claims that "in this case, improved useful knowledge was applied to a service industry rather than a physical process, and with considerable success" (*Enlightened Economy*, 231).

material on early financial economics is provided by Geoffrey Poitras.[27] The collective volume edited by William N. Goetzmann and K. Geert Rouwenhorst contains excellent insights.[28] We will make a critical use of the older work by Gabriel A. Hawawini and Ashok Vora on interest extraction techniques.[29] Regarding international dimensions, we mention (in addition to the work already quoted by Zemon Davis) the work of Christopher Lewin and Margaret De Valois on actuarial tables and compound interest as well as Bertram Schefold's analysis of Leibniz's contribution to actuarial calculus.[30]

Long-Term Productivity Gains in Financial Calculus

We begin with a canonical statement of the annuity problem. Its emergence as the central problem of finance at the time of the Commercial Revolution in the twelfth and thirteenth centuries is associated with mathematician Leonardo of Pisa (known as Fibonacci), who drew on Arab science. Considering an annuity, Fibonacci showed that the interest rate could be formalized as the implicit solution of a mathematical function.[31]

Fast forward, this formalism crossed centuries and became ubiquitous. Box 1 shows a selection of statements of the problem drawn from the principal contributions of the classic age. Using the formalism proposed by Francis Baily in the *Doctrine of Interest*, which subsequently set the standard, we have *a*, the price paid to purchase an annuity that yields one unit of currency per year, *n*, the number of years, and *i*, the interest rate to be discovered. The value of this unit annuity is:

$$a=\sum_{t=1}^{t=n}\frac{1}{(1+i)^t} \tag{1}$$

Or using the properties of sequences:

$$a=\frac{1-(1+i)^{-n}}{i} \tag{2}$$

[27] Poitras, *Early History of Financial Economics*.

[28] Goetzmann and Rouwenhorst, *Origins of Value*.

[29] Hawawini and Vora, "Yield Approximations"; Hawawini and Vora, *History of Interest Approximations*.

[30] Lewin and De Valois, "History of Actuarial Tables"; Lewin, "Emergence of Compound Interest"; Schefold, "From Kaspar Klock's 'De Aerario'"; Schefold, "Gottfried Leibniz."

[31] Goetzmann, "Fibonacci"; Fibonacci, *Liber Abaci*.

Box 1 The changing face of the present value equation, 1677–1882

The way the present value equation has been formulated underwent significant changes over time. We reproduce here a selection of formulations, drawn from the works cited in the text.

1. Michael Dary, *Interest Epitomized*, 1677, p. 7:

$$r=\frac{p+n}{p}-\frac{n}{r(t)\times p}$$

Dary did not superscript the exponent t, which means that his $r(t)$ should read r^t.

2. Edmund Halley, "Compound Interest," 1706, p. 33:

$$\frac{a}{z}=\frac{z+a}{z}r^{t} - {}_{r}{}^{t+1}$$

Due to its strange alignment Halley's r^{t+1} above was interpreted by later editors to mean $(t+1)/r$. See later editions of Sherwin's *Mathematical Tables* (see fourth edition of 1761, p. 20); Institute of Actuaries (reedition of Halley, 1861, p. 266).

3. John Ward, *Clavis Usurae*, 1710, p. 85:

$$\frac{U}{P}=\frac{U}{P}R^{t}+R^{t}-R^{t+1}$$

4. Thomas Simpson, *Treatise on Algebra*, 1745, p. 209:

$$R^{n+1}-\overline{\frac{A}{v}+1}\times R^{n}+\frac{A}{v}=0.$$

5. Francis Baily, *The Doctrine of Interest and Annuities*, 1808, p. 52:

$$p=a.\frac{1-(1+\rho)^{-n}}{\rho}$$

6. William Sutton, *Institute of Actuaries' Textbook*, 1882, p. 27:

$$a_{\overline{n|}}=\frac{1-v^{n}}{i}$$

Notation correspondence for the aforementioned equations:

Meaning	Own notations	Dary	Halley	Ward	Simpson	Baily	Sutton
Annuity (yearly income flow)	1 (unit annuity)	u	a	U	A	a	1 (unit annuity)
Present value (price)	a	p	z	P	V	p	$a_{\overline{n}\vert}$
Interest rate	i	$r-1$	$r-1$	$R-1$	$R-1$	ρ	i
Time	n	t	t	t	n	n	n

Note: Until the early nineteenth century, the interest variable designated the growth factor, or (1 + i). For example, in Simpson's equation, our i corresponds to R – 1. The modern way of formulating the problem in such a way that the unknown is the interest rate was first established by Francis Baily (1808).

Equation (2) is the annuity problem, described by Baily as "so involved, that [the] true value [of the interest rate] cannot be expressed by any of the known rules of algebra."[32] Equation (2) only defines yield i implicitly: Its *actual* computation—the extraction of the root—raises numerical challenges. In the absence of any computing support, it could only be sorted by what previous authors describe as "brute force" . . . . or by cunning and the development of a computing infrastructure, as we shall see.

Vernacular Techniques

Numerical resolution of the annuity problem was originally a work of craft, patience, and inspiration, a sophisticated art that exploited in a clever fashion certain properties of the problem to generate heuristic solutions. Bearing witness to the effervescent climate of early modern England's numerical culture, such techniques were described in popular guides and almanacs

[32] Baily, *Doctrine of Interest*, 172. Technically, this is the case for n > 4.

that provided readers with guidance on the annuity problem.[33] The extent to which this knowledge impregnated the trading world and beyond is actually staggering.

The most widespread approach was by trial and error, also known as the False Position. It was a *tâtonnement* process, an algorithm sans theory: Assume a yield, plug it in equation (2), then observe whether the number thus calculated corresponds to the annuity's price (the "purchase money"). If it was inferior (superior) to the annuity's actual price, the assumed yield had been set too high (low). As John Ward explained in 1710, the calculation could then be repeated with a lower (higher) yield until one was satisfied.[34] Calculators could also resort to the so-called Double False Position, which assumed two yields, respectively larger and smaller than the true value, and interpolated.[35]

A fundamental tool was the annuity tables (see fig. 1). Technically, they tabulated equation (2). That is, they gave the present value of unit annuities as a function of common ranges of interest rates and maturities.[36] It was thus possible to read the annuity table "backward" to bracket the interest rate solution: Given the present value of the annuity and the maturity, the table enabled determining an upper bound and a lower bound for the interest rate.[37] In other words, the annuity tables helped quickly identify a ballpark within which the solution lay, making it possible to take a guess which could then be refined with the help of the false position.[38]

[33] Otis, *By the Numbers*. Discussing the false position, Gavin and Schärlig, *Longtemps avant l'algèbre*, note that it was used to solve all kinds of real-life problems.

[34] Ward, *Clavis Usurae*, 85–86.

[35] Newton, *Scale of Interest*, 103–4; Jeake, *Compleat Body of Arithmetic*, 496–505; Martindale, *Country-Survey-Book*, 137–39. Strictly speaking, the trial-and-error method might not be classified among algorithms, as defined by Schneider and Gersting, *Invitation to Computer Science*, who emphasize unambiguity as a defining feature of the latter. For instance, trial-and-error methods did not guide the calculator in his choice of assumed yield and they did not give indications as to when to stop, though of course conventions existed.

[36] Annuity tables were derivatives of the interest tables which gave the present value of future payments, given the interest. The former were obtained by summing the present value of one unit of currency over j years, j belonging to $[1, n]$. Flemish mathematician Simon Stevin, supposed inventor of the decimal fraction, is credited with having published in 1582 (in Dutch) the first annuity table. The tables were reprinted in French in 1585 as part of his magnum opus, *l'Arithmétique*, 763–83. Lewin, "Emergence of Compound Interest," on early modern annuity tables; Deringer, "Mr. Aecroid's Tables," provides a case study.

[37] See note to fig. 1.

[38] Kersey, *Mr Wingate's Arithmetick*, 380–81 for the table, and 398–400 for the instructions on how to use it to find the rate by interpolation.

FIGURE 1 ◆ EXTRACT FROM JOHN SMART'S TABLE (1726)

The Fourth Table of Compound Intereſt.

The Preſent Value of One Pound *per Annum* for any Number of Years to come, &c.

Years.	At 2 *per Cent.*	$2\frac{1}{2}$ *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Cent.*	4 *per Cent.*	$4\frac{1}{2}$ *per Cent.*	*Years.*
$25\frac{1}{2}$	19.8237,2533	18.6891,1802	17.6467,0832	16.6876,9148	15.8041,9280	14.9891,6203	$25\frac{1}{2}$
26	20.1210,3576	18.9506,1114	17.8768,4239	16.8903,5226	15.9827,6918	15.1466,1145	26
$26\frac{1}{2}$	20.4154,1699	19.2088,9563	18.1036,0031	17.0895,5698	16.1578,7769	15.3006,3352	$26\frac{1}{2}$
27	20.7068,9780	19.4640,1087	18.3270,3145	17.2853,6450	16.3295,8575	15.4513,0282	27
$27\frac{1}{2}$	20.9955,0686	19.7159,9564	18.5471,8477	17.4778,3283	16.4979,5932	15.5986,9236	$27\frac{1}{2}$
28	21.2812,7235	19.9648,8865	18.7641,0820	17.6670,1884	16.6630,6322	15.7428,7351	28
$28\frac{1}{2}$	21.5642,2241	20.2107,2745	18.9778,4929	17.8529,7858	16.8249,6088	15.8839,1614	$28\frac{1}{2}$
29	21.8443,8466	20.4535,4991	19.1884,5456	18.0357,6700	16.9837,1464	16.0218,8853	29
$29\frac{1}{2}$	22.1217,8667	20.6933,9263	19.3959,7019	18.2154,3825	17.1393,8546	16.1568,5755	$29\frac{1}{2}$
30	22.3964,5555	20.9302,9259	19.6004,4132	18.3920,4541	17.2920,3330	16.2888,8854	30

Source: Smart, *Tables of Interest*, p. 78.
Note: Consider the problem of determining the yield for a 28-year annuity purchased for £18. The exact four decimals solution is 3.3435%. Using Smart's table, the early modern computer would have found that £18 was bracketed by £18.7641 (the price of a 28-year annuity at 3%) and £17.6670 (the price of a 28-year annuity at 3.5%), ergo the true yield belonged to the [3%,3.5%] interval.

It is important to note that these rough-and-ready techniques persisted well after the appearance of the more advanced algebraic methods that we will discuss in the next subsection, likely because they were not so rough. Throughout the eighteenth century, we see popular manuals recommending them.[39] Purists expressed frustrations and described this trial and error as a vile method. Writing in 1808, Francis Baily would feel the need to attack John Ward's still popular empirical treatise, which had advocated in favor of trial and error.[40]

We would be ill advised to oppose subaltern algebra to the highbrow variety. In fact, more than once, a trick developed by practical computers would theorize retrospectively, leading to the articulation of a "sophisticated" method. Even Baily admitted the usefulness of annuity tables. As he noted, it was highly desirable to always begin by looking at one's table. First, one might get lucky, if for instance the solution was close to a table's precalculated solutions. If that was not the case, at least linear interpolation from the table would put the computer on the right track.[41] In fact, even after the spread of algorithms, annuity tables would retain a significant role as providers of starting values.

Reflecting this, the transcription of a discussion that took place at the Institute of Actuaries in 1875 shows one Andrew Baden asserting that there were "two ways of going to work to approximate to the true rate of interest—a scientific and an unscientific way." He supported the unscientific one, which consisted in "making a series of direct trials for the rate." It worked well, Baden insisted, because "you acquire by practice a tact in guessing the proper quantity to interpolate in the tables you use." The chairman of the meeting suggested that this "guessing power" accrued from Baden having "worked out a great many cases." Could it be said he was not "guessing" but "using knowledge previously obtained"?[42]

[39] Bertrand, *Développement nouveau*, 1:668; Keith, *Complete Practical Arithmetician*, 264–72.

[40] Baily, *Doctrine of Interest*, vii. As Baily put it, mathematical formulae appear to have been "beyond Ward's capacity, as he still persisted in the old method of finding the value by Trial and Error".

[41] Baily, *Purchasing and Renewing*, 1802, 19–24.

[42] Sutton, "Rate of Interest," 88–90. For further evidence of the survival until the late nineteenth century of interpolation as a method to approximate the yield of an annuity, see McKenzie, "Method of Approximating"; Oakes, *Finding the Intermediate Rates*; Todhunter, *Institute of Actuaries' Text-Book*, 115–21.

The Mathematics of the Annuity

Starting in the late seventeenth century, at about the same time when the first green shoots of the Financial Revolution were showing their heads, mathematicians of considerable standing, including names such as Isaac Newton, Edmund Halley, or Thomas Simpson, started developing formulae that would help mechanize the resolution of the annuity problem. In this subsection we review how these formulae operated. To that end, we take advantage of mathematician Augustus de Morgan's mid-nineteenth-century compilation and expand it by going over the historical annuity literature with a fine-toothed comb.[43] In this fashion, we construct what is to our knowledge the most comprehensive catalog of the formulae available to calculators between 1680 and 1880 ever put together (table 1).

As we discovered, formulae came in two species, corresponding to two main approaches. The first species is formed of *direct formulae* enabling calculators to obtain an *approximate* solution to the annuity problem. They worked like this: The parameters of the problem were fed into an equation, which returned an approximation—a solution close enough to the true interest rate. The second species supported an algorithmic approach that solved the annuity problem recursively and *exactly*. The formulae were *update functions*. Here, inputs were a trial interest rate solution along the maturity and the price of the annuity. The output was another interest rate, closer to the true value. This new interest rate could in turn be input into the same update function to yield an even better solution, and so on. Describing this in the language of his time, Francis Baily spoke of this process as consisting of "approximating [the] true value [. . .] to any degree of exactness."[44]

The upper part of table 1 lists the direct formulae (approximations). We identify a total of eight. Four were put forward before 1800: by Isaac

[43] De Morgan, "Determination of the Rate of Interest."

[44] Baily, *Doctrine of Interest*, 114. The difference between these two approaches has not been properly understood previously. Hawawini and Vora argue incorrectly that in practice there only existed approximate formulae, the reason being, they allege, the low quality of existing algorithms and the absence of "modern computers." They state that "today calculators and computers can solve this problem by a rapid series of iterations" ("Yield Approximations," 145). A similar misconception is found in Mauro, Sussman, and Yafeh, who state incorrectly that "exact calculation [of yields] were only developed in the twentieth century, and their application often requires the use of calculators and computers" (*Emerging Markets*, 41–42).

Newton (1670),[45] Edmund Halley (1706),[46] and two by Thomas Simpson (1745).[47] Three were put forward later, by exchequer court judge "Baron" Francis Maseres (in 1804), by stockbroker Francis Baily (in 1808), and by Augustus de Morgan (in 1859).[48]

The lower part of the table lists the update functions (algorithms). A total of nine were culled up of which three were delivered before 1800: by mathematician Michael Dary (1677),[49] by Halley (in 1706), and by Simpson (in 1745), who offered them alongside their proposed approximate formulae.[50] Note that Simpson's update function was given in the shape of "cooking directions." As we discovered however, following the instructions with a pencil at hand lands on an update function that would be given as "new" by Maseres in 1804.[51]

Not counting Maseres's update function, since it was Simpson's, we identify six individual update functions after 1800. Baily's update function given in 1808, along with his interest rate approximation, deserves a special emphasis because it has been celebrated in subsequent works and contributed mightily to a view of Baily as the father of "modern" financial mathematics.[52] Its efficiency was further enhanced (it is said) by a small

[45] Turnbull, *Correspondence of Isaac Newton*, 1:24–26. On Newton's formula, see note to table 1.

[46] Halley, "Compound Interest," 34.

[47] The first formula Simpson proposes in the first edition of his *Treatise* ("Thomas Simpson (1)" in table 1) is not reproduced in the subsequent editions. As Simpson noted, formula (1) was heavier to use, but only marginally more precise. Applying a numerical example Simpson himself proposes, to which we will later come back, formula (1) yields a rate deviating by 0.004 basis points from the true rate, while formula (2) deviates by 0.007. In addition, formula (1) tended to break down entirely for certain ranges of the parameter space (especially high interest rates). In contrast, formula (2) was more robust, as it consistently produced rates reasonably close to the true value. Formula (2)'s superior versatility rendered it better suited for advertisement; it could be mobilized in a broader variety of practical annuity cases. See Simpson, *Treatise of Algebra*, 1745, 213.

[48] Maseres, "Some Difficult Passages," 337; Baily, *Doctrine of Interest*, 127; De Morgan, "Determination of the Rate of Interest," 65.

[49] Dary, *Interest Epitomized*, 7. De Morgan credits it with being the first algorithm ever designed to solve the annuity problem ("Determination of the Rate of Interest," 63).

[50] Halley, "Compound Interest," 33. Note that Halley viewed his algorithm as complementary to his direct formula. The latter was to be applied to annuities with durations shorter than 40 years. For longer maturities, Halley gave a direct formula that provided a starting value for his algorithm (see the notes to table 1).

[51] Simpson, *Treatise of Algebra*, 1745, 149; Maseres, "Some Difficult Passages," 294–96. Maseres compares "his" formula to Halley's and makes no reference to Simpson's, which he may have ignored.

[52] Baily, *Doctrine of Interest*, 127–32. See the classic textbook accounts of Sutton, *Institute of Actuaries' Text-Book*, 81–85; King, *Theory of Finance*, 1882, 212–15; Todhunter, *Institute of Actuar-*

substitution which Baily's protégé, "computer" George Barrett, proposed in 1811.[53] Finally, a miniboom in update functions occurred during the Victorian era, when four more were crafted. They were due respectively to Augustus De Morgan (1859), J. J. McLauchlan (1874), and George Hardy (1882), who proposed two.[54]

The comparatively more intense production of update functions after 1800 (six compared to three direct formulae) speaks of a greater interest in the former. The pivotal point, from this respect, is the "Maseres-Baily moment." While these two authors volunteered both direct formulae and update functions, they displayed a strong support for algorithms. This was a new attitude. Both speak of a hierarchy in which algorithms would be the new rulers. Maseres viewed direct formulae as subordinate, providing "groundwork or basis" for algorithms, by which he meant approximate formulae generated starting values.[55] Baily, for his part, spoke of algorithms as the "principal" approach.[56]

In fact, in the wake of the publication of Baily's treatise, this change of mood became evident. In 1815, financial mathematician Joshua Milne declared that he did not care to introduce any other method than the algorithm. He insisted that the combined use of tables (which gave starting values) and algorithms enabled determination of the true interest rate "to a degree of exactness abundantly sufficient for any useful purpose."[57] In 1839, actuary Peter Hardy doubled down, deeming direct formulae "complicated in their form, and laborious in their application."[58] By the end of the period, approximate formulae were looked at as antiquities—*curiosum* for experts and training material for students.[59]

Logically enough, this tendency can also be read in the effervescence that characterized the production of annuity tables. Enhancements in the quality of tables—especially tables with smaller steps—were important because this increased the performance of algorithms. The reason is intuitive

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ies' Text-Book, 110–12.

[53] Knowledge of Barrett's improvement, however, only surfaced following the death of Baily, when De Morgan, who was mining Baily's papers, stumbled on it and published it. See De Morgan, "Account of a Correspondence," 189.

[54] De Morgan, "Determination of the Rate of Interest"; McLauchlan, "Formulas for the Approximate Determination"; Hardy, "Rate of Interest in Annuities-Certain."

[55] Maseres, "Some Difficult Passages," 354.

[56] Baily, *Doctrine of Interest*, 115.

[57] Milne, *Valuation of Annuities*, 686–87.

[58] Hardy, *Doctrine of Simple and Compound Interest*, 48.

[59] King, *Theory of Finance*, 1882, 116; Todhunter, *Institute of Actuaries' Text-Book*, 105–6.

enough: If one shoots from a closer range, one tends to take better shots. A standard consisting of 100 basis points steps (thus going from 2% to 3%, 3% to 4%, etc.) had been set by Simon Stevin in 1582.[60] It had been superseded in 1726 by John Smart's more granular *Tables of Interest*, adopting the same norm except for the 2% to 5% range, where it provided 50 basis points steps. Smart's tables were republished in 1736 and 1780. In 1808, Baily still appended them to his *Doctrine of Interest*, while calling for their improvement. Since Baily was advocating for algorithms, he also urged convenient access to quality starting values.[61]

Smart's tables were still reproduced in Milne's *Valuation of Annuities* (1815). Ten years later, Francis Corbaux's tables ushered in a high granularity mania. Corbaux's tables pioneered steps of 25 basis points, but they covered the 3% to 6% range only.[62] In 1839, Peter Hardy's tables ushered in 25 basis points steps for the 0.25% to 5% range.[63] In 1852, Thomas George Rance implemented this new standard over the full 0.25% to 10% range.[64] Finally, in 1877, Lieutenant Colonel W. H. Oakes introduced steps of 12.5 basis points, a nirvana of sorts.[65] The features of these respective tables are presented in the appendix.

Horse Races

Having clarified the meaning and operation of the computing landscape, we now proceed with measuring accuracy in financial calculus. In what follows we seek to measure the precision achieved by a computer using the best available formula to sort out the annuity problem at any point in time. In other words, we trace the progress of the efficiency frontier. One question

[60] Stevin, *Tafelen van Interest*, 35–50. The first column gives the present value of 1 currency unit paid at increasingly more remote horizons; the second column provides for each line j the sum of the values in the first column between line 1 and line j, which generates an annuity table. On Stevin's tables, see n. 36.

[61] Baily complained that Smart's work "contain[ed] so many errors that it cannot be depended upon" and justified the publication by saying that he was giving them "to the world 'with all their imperfections on their head'" because he had "neither time nor inclination to calculate them anew" (*Doctrine of Interest*, vii). Baily then went on subsidizing the work of calculator Barrett, possibly in the hope that the tables would be replaced by something better. On the relation between Baily and Barrett, see De Morgan, "Account of a Correspondence."

[62] Corbaux, *Doctrine of Compound Interest*.

[63] Hardy, *Doctrine of Simple and Compound Interest*.

[64] Rance, *Tables of Compound Interest*.

[65] Oakes, praising the higher granularity of his tables (*Tables of Compound Interest*, iii); also splashed in the title ("by Eighths of a Percentage Point").

we ask is, when did algorithms start dominating direct formulae, if this is indeed what happened? We are also interested in building an image of long-term gains in computing productivity. Beyond this, the question we aim at is, what factors account for changes in computing efficiency? To answer these questions, we adopt the vantage point of a contemporary calculator and, using a spreadsheet, we replicate the calculations and measure their performance.

The general idea is that the larger the computing error, the less efficient is the method. We posit that every formula identified above was available to an ideal calculator whose travails we replicate.[66] We consider, too that labor input is constant: each formula involves roughly the same amount of computing work. This means that if a formula is more *precise* than its predecessor, it is more *efficient*. This approach is consistent with contemporary commentary comparing formulae without reference to computing effort.[67] Indeed, novel formulae typically contributed an accuracy improvement that was of an order of magnitude compared to the best formula extant, while complexity of the calculation remained comparable. The exception that proves the rule is the two update functions by George Hardy in 1882. They were less accurate than the best formulae extant at the time, but they had attractive labor-saving properties. This unusual feature was emphasized by observers.[68]

Long-term efficiency gains were estimated by generating 1,000 random annuity problems whose solutions spanned the range of relevant values (2%–10%). Since the performance of alternative formulae vary with the term of the annuity, we identified three benchmarks, namely 15, 30, and 60 year contracts.[69] For each random problem, we computed (a) the "true" solution and (b) the solutions generated by each formula.[70] We then calculated the *distance* between the exact solutions and the output from the

[66] This was not necessarily always the case. Simpson's cooking directions in 1745 and Barrett's formula of 1811 both show there could be a long lag between discovery and publication.

[67] Baily, *Doctrine of Interest*, 120, 127.

[68] Hardy, "Rate of Interest in Annuities-Certain," 267, 271; King, *Theory of Finance*, 1898, 41–43. Note that the loss of precision which the use of Hardy's formulae involved was hardly dramatic: assuming an annuity was purchased for £10 and paying £1 for 15 years, Hardy's second formula found the true rate at five decimals' precision, against six decimals for McLauchlan.

[69] Halley, "Compound Interest," 33; Simpson, *Treatise of Algebra*, 1745, 213; Baily, *Doctrine of Interest*, 132, all provided evidence of awareness that, for instance, direct formulae in vogue in the eighteenth century underperformed for longer maturities (and higher interest rates).

[70] De Morgan's direct formula was the only one whose output we could not replicate. As a result, it was excluded from the exercise. We hope to be able to resolve this quandary in the future.

formula. The quality of each individual formula is measured by the median distance.[71]

To "normalize" computing effort, the solution associated with each individual formula for a given horizon and random problem was computed after exactly one operation, consisting either in the application of an approximate formula or (in the case of algorithms) in the application of the update function to starting values read in an interest table.[72] This amounts to neglecting the time and effort spent on retrieving starting values. As we learned for ourselves however, a trained calculator can execute the task in less than a minute. We identify six significant tables: Stevin (1582), Smart (1726), Corbaux (1825), Hardy (1839), Rance (1852) and Oakes (1877).[73]

Software, Hardware

Figure 2 distills the results, distinguishing between the three different maturities mentioned (15, 30, and 60 years). For each, we propose two charts. The left-hand chart displays the *performance* of every individual formula identified. By showing the precision secured depending on the table's granularity, left-hand charts also visualize the impact of annuity tables on accuracy. To avoid crowding the picture, we show the accuracy associated to two granularity levels only, even as we identified six major breakthroughs. Specifically, we present performance associated with the coarsest (Stevin) and finest (Oakes) granularity. Figures are given in logarithms: Since we are dealing with interest rates, an error of 0.001 means an error of 10 basis points (calculating 5.10% instead of 5%).

The three left-hand charts exhibit, first, a striking tendency for new formulae to generate more precise results. This speaks of a secular pursuit of greater accuracy. Note in passing that the results also bear witness to the peculiar standing of Hardy's two formulae which, as already stated, were an attempt at limiting labor costs. As a result, they come across as

[71] We prefer this to the obvious alternative (the standard error), because contemporaries spoke in terms of absolute errors. The point is not material, however, because overall results are not affected by this convention.

[72] Milne, *Valuation of Annuities*, 686–87, draws the parallel between applying a direct formula and running one iteration of the algorithm after securing a starting value from the annuity table.

[73] Note that some are reconstructed. For instance, Francis Corbaux's table only covered the range 3% to 6%. This means that in 1825, when Corbaux's table was printed, an efficient calculator had to resort simultaneously to it and, where Corbaux's table provided no coverage, to what was at this point the most granular table available (Smart's). In our simulation, "Corbaux's table" is really a patchwork of Corbaux and Smart. See notes to fig. 2 for details.

regression. The other striking feature of left-hand charts is the massive effect of granularity on accuracy. The precision improvement secured by swapping Stevin's tables for Oakes's ranges is between one and three orders of magnitude, depending on the horizon and update function mobilized.

The right-hand side charts, by contrast, display the time profile of efficiency gains. They are constructed by imagining that at every point in time, the calculator uses the best available resources given the chosen method: If they use approximations, then they use the best approximate formula extant. If they use the algorithm, then they use the best update function extant, coupled with the best annuity table available. A corollary is that, if a new, superior table becomes available, the accuracy of the best available method receives a boost. The publication of a table with superior granularity thus registers as a positive technological shock, just like the design of a new, superior update function.

The charts bring home important messages. They speak of a hidden story of secular numerical progress, long before the advent of modern computers. The median error incurred from the best method went down from 10 basis points in round numbers in the seventeenth century (which was not that bad!) to one *hundred thousandth* of a basis point in the high Victorian era (!!). In addition, the charts underscore that both the seventeenth- and eighteenth-century focus on approximate formulae and the nineteenth-century focus on algorithms can be rationalized as the result of the superior performance of the respective technology. Approximate formulae dominated as long as they were good enough. They became obsolete after new, more powerful update functions helped entrench the notion that higher precision was within reach.

In fact, while the power of approximate formulae plateaued in the late eighteenth century, algorithms took off. They reached enormous precision during the Victorian age. At broad brush, the pivot is the Maseres-Baily moment, precisely when algorithms started being hailed as the future of computing. At that point (circa 1800) however, their superiority was hardly overwhelming for they only had an edge in the case of long annuities. As for Baily's own formula, it does not come across as revolutionary. In fact, if we look closer, we find that a significant part of the edge which algorithms enjoyed for long-dated annuities had been realized since Simpson's "cooking directions" of 1745—except that, as we said, the innovation had remained an insider thing, to the point that Maseres could say in 1804 that he had discovered the formula.

TABLE 1 ◆ A LIST OF FORMULAE TO DETERMINE THE RATE OF INTEREST OF A "TERMINABLE" (I.E., FIXED-TERM) ANNUITY (1670–1882)

Author	Type	Date	Formula
Isaac Newton	Direct formula (See note 1 below)	1670	$\log(1+i)=\dfrac{6\log\left(\frac{n}{a}\right)}{100-50\log\left(\frac{n}{a}\right)}$
Edmond Halley	Direct formula, for n < 40	1706	$i=y-(y^2-2y\beta)^{\frac{1}{2}}$ With: $\beta=\left(\frac{n}{a}\right)^{\frac{2}{n+1}}-1$ $y=\dfrac{6}{n-1}$
Thomas Simpson (1)	Direct formula	1745	$i=(H^2+K)^{\frac{1}{2}}-H$ With: $H=\dfrac{f}{g}$ $K=\dfrac{q}{f}$ $q=\dfrac{2n-2a}{n(n+1)}$ $f=\dfrac{n+8}{15}-\dfrac{(n+2)(n+3)q}{20}$ $g=\dfrac{1}{2}-\dfrac{(n+3)q}{5}$
Thomas Simpson (2)	Direct formula	1745	$i=\dfrac{30Q-(2n+1)4}{6Q(5Q-3n-4)+\frac{1}{6}(11n+13)(n+2)}$ With: $Q=\dfrac{n(n+1)}{2(n-a)}$

(Continued)

TABLE 1 ◆ (*Continued*)

Author	Type	Date	Formula
Francis Maseres	Direct formula	1804	$i=\theta+b\theta^2+\frac{\lambda^2}{\lambda-k}$ With: $\theta=\frac{2n-2a}{n^2+n}$ $\lambda=(2b^2-c)\theta^3$ $k=(5b^3-5bc+d)\theta^4$ $b=\frac{n+2}{3}$ $c=b\frac{n+3}{4}$ $d=c\frac{n+4}{5}$
Francis Baily	Direct formula	1808	$i=\frac{(12-(n-1)\beta)\beta}{12-2(n-1)\beta}$ With: $\beta=\left(\frac{n}{a}\right)^{\frac{2}{n+1}}-1$
Augustus de Morgan	Direct formula	1859	$\log(1+i)=\alpha\frac{\log(n)-\log(a)}{n+1}$ With: $\alpha=1+\frac{1}{1-t}-\frac{1}{20}\frac{t^3(1+t)}{(1-t)t^2}$ $t=\frac{1}{3}\frac{n-1}{n+1}\frac{\log(n)-\log(a)}{0.4342945}$
Michael Dary	Update function	1677	$i=\frac{1-(1+j)^{-n}}{a}$
Edmund Halley	Update function, for n < 40 (See note 2)	1706	$i=\frac{1}{a+\frac{1}{j(1+j)^n}}$

(*Continued*)

TABLE 1 ◆ (*Continued*)

Author	Type	Date	Formula
Thomas Simpson/ Francis Maseres	Update function	1745/ 1804	$i = j + \dfrac{(a^{-1}+1)(1+j)^n - a^{-1} - (1+j)^{n+1}}{(n+1)(1+j)^n - n(a^{-1}+1)(1+j)^{n-1}}$
Francis Baily	Update function	1808	$i = j + \dfrac{j(a_1 - a)}{a - nv^{n+1}}$
George Barrett	Update function	1811	$i = j + \dfrac{j(a_1 - a)}{a_1 - nv^{n+1}}$
Augustus de Morgan	Update function	1859	$i = j + \dfrac{j(a_1 - a)}{a - nv^{n+1} + \dfrac{n(n+1)}{2}\dfrac{j(a_1 - a)v^{n+2}}{a - nv^{n+1}}}$
J. J. McLauchlan	Update function	1874	$i = j + \dfrac{j(a_1 - a)}{a - nv^{n+1} + \dfrac{n(n+1)}{2}\dfrac{j(a_1 - a)v^{n+2}}{a_1 - nv^{n+1}}}$
George Hardy (1)	Update function	1882	$i = j + h\left(\dfrac{\Delta a_1 - \frac{1}{2}\Delta^2}{a - a_1} + \dfrac{\frac{1}{2}\Delta^2}{\Delta a_1 - \frac{1}{2}\Delta^2}\right)^{-1}$ With: $\Delta^2 = \Delta a_2 - \Delta a_1$; $\Delta a_1 = a_2 - a_1$; $\Delta a_2 = a_3 - a_2$
George Hardy (2)	Update function	1882	$i = j + h\left(\dfrac{\Delta a_1 - \frac{1}{2}\Delta^2}{\dfrac{(\Delta a_1)^2}{a - a_1} + \frac{1}{2}\Delta^2}\right)$

Notations:

The formulae we reproduce here are written following the actuarial notations as they ended up crystallizing toward the end of the nineteenth century (except for the unfamiliar actuarial symbol subscripted and enclosed in a right angle, denoting a fixed-term maturity appearing in Sutton's equation in box 1, which we neglect here for the sake of simplification).

- i : The interest rate to be found (the unknown).
- a : Present value of future income stream (the price to be paid to buy an income stream of £1 per year for the period n, or "purchase money").

(*Continued*)

TABLE 1 • (*Continued*)

- a_1: Closest annuity value from a, found in the annuity table.
- a_2: Lower bound of a_1, found in the annuity table.
- a_3: Lower bound of a_2, found in the annuity table.
- h: Granularity of the annuity table (e.g., for a table with increments of 0.5%, then $h = 0.005$).
- j: Trial rate, taken in the table, corresponding to the annuity value a_1.
- n: Number of years (maturity of the annuity).
- $v = (1 + j)^{-1}$.

Notes:

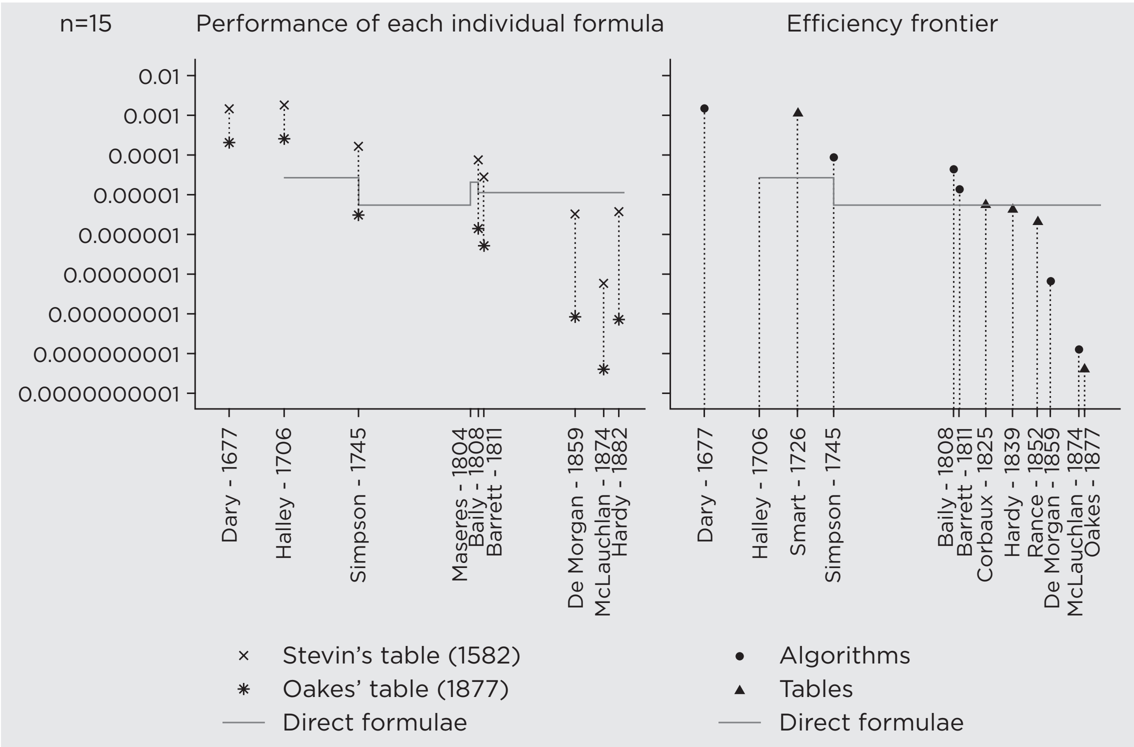
1) Isaac Newton's (1670) direct formula was specifically calibrated for the problem he was tasked to solve. It had no generality. A generalized version of Isaac Newton's formula was proposed almost two centuries later by De Morgan, "Determination of the Rate of Interest," p. 65. Hawawini and Vora, "Yield Approximations," pp. 148–49, attribute it mistakenly to one John Newton. See Brief, "Yield Approximations," himself mistakenly stating that the formula appeared in the third volume of Isaac Newton's correspondence, when it appeared in the first.

2) For the first iteration of his algorithm (meant for annuities of maturity greater than 40), Halley did not recommend taking the trial rate from an annuity table but rather proposed a formula to calculate it directly (*Compound Interest*, p. 33): $j = 1/a - (1/a) \cdot (a/(1+a))^n$. Halley claimed that the result of this first iteration would approach "the true rate sufficiently." Of course, Halley pointed out that "if greater accuracy be desired," the process could be repeated.

Right-hand charts also demonstrate the vital complementarity between tables and algorithms. Tables were not only an important tool for vernacular techniques. Looking at the first half of the nineteenth century, we see that granularity improvements drove efficiency gains. Between Barrett's improvement of Baily's formula in 1811 and De Morgan's novel update function in 1859, incremental accuracy gains were due to table enhancements: Corbaux in 1825, Hardy in 1839, and Rance in 1852 register as positive shocks whose cumulative effects were of an order of magnitude. So, while considerable increases in computing efficiency were achieved by relying on Baily's "software" (or Barrett's tweak of Baily's software), the force that powered this—at least until 1852—was the improvement of "hardware."

To give a sense of the remarkable precision achieved, we might in conclusion place ourselves in 1880, at the end of the period. Consider a computer seeking to determine the yield for a 15-year unit annuity purchased for £10.

FIGURE 2 ◆ ABSOLUTE VALUE OF THE DIFFERENCE BETWEEN THE EXACT SOLUTION AND THE FORMULAE'S SOLUTIONS, FOR 1,000 RANDOMLY GENERATED ANNUITY PROBLEMS



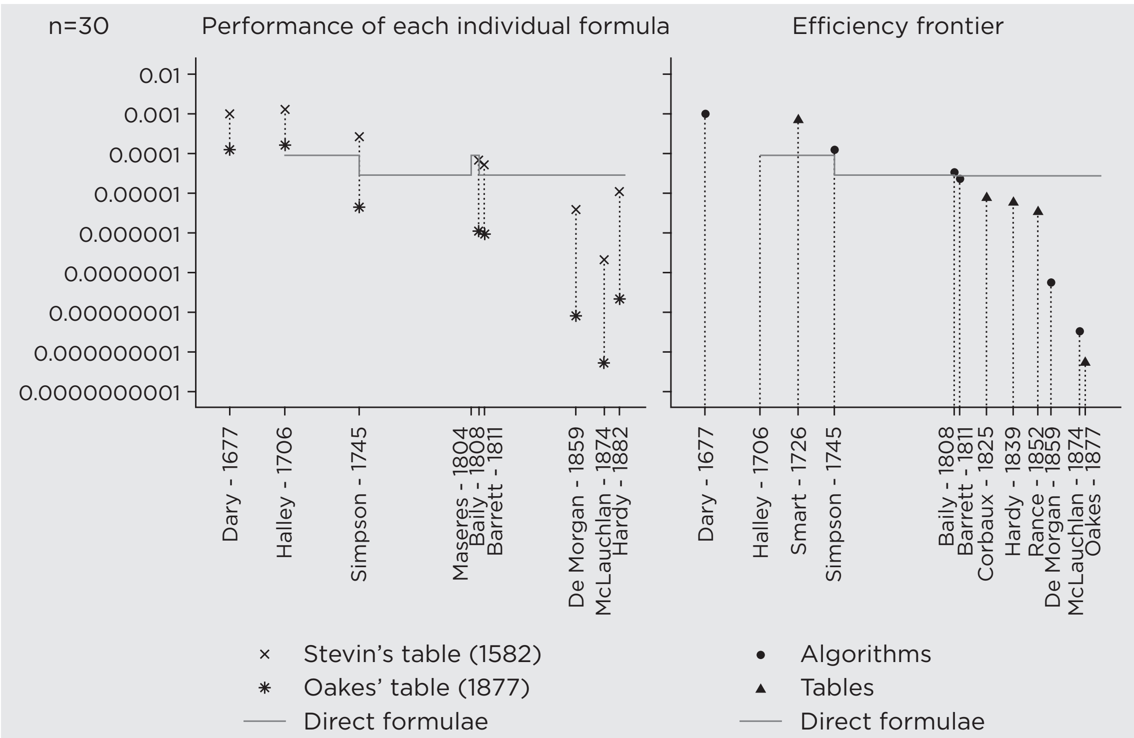
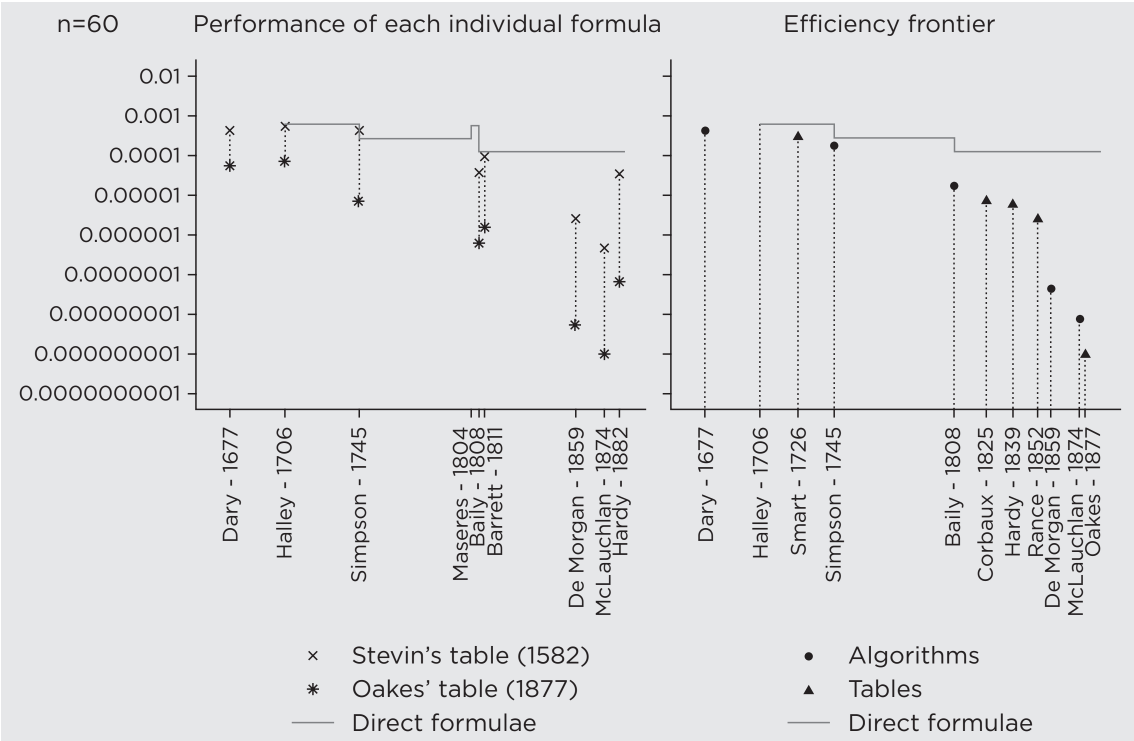


FIGURE 2 ◆ (*Continued*)



Source: Authors' simulation.

Notes:

1) When an author proposed two formulae, we took the more precise of the two: we took Simpson (1) rather than Simpson (2), and Hardy (2) rather than Hardy (1).
2) As Stevin's table only covers maturities up to 30 years, we constructed for n = 60 a "synthetic Stevin's table," that is, an annuity table for a maturity of 60 years, with otherwise all features of Stevin's table.
3) For the right-hand charts: (a) The output of Dary's algorithm is calculated using Stevin's table. (b) As Corbaux's table only covered an interest rate range of 3%–6%, we created a synthetic Corbaux's table, filling in the gaps by using Smart's granularity for 2%–2.5% and 7%–10%. In other words, we assumed that calculators had access to both Smart's and Corbaux's tables and looked at either of them depending on the interest rate problem they were confronted with. (c) We followed the same logic to create a synthetic Hardy's table, with granularity of 25 basis points up to 6%, and 100 basis points up to 10%.

With 6-digit precision, the "true" solution is 5.556497%. Using Oakes's table, the Victorian analogue machine would determine that the solution was in the [5.5%-5.625%] interval. Feeding 5.5% (the closest bound) into McLauchlan's formula, it would then return in fact 5.556497%.

The Republic of Numbers (1680–1800)

We now move to the examination of the successive orders that structured the accumulation of computing power. This section deals with the age of the "mathematicians of wealth." These scientists helped sort the massive market for leases, a type of annuity contract providing the cornerstone of English property. They did it by lending their talents and prestige to the design of formulae, but also acted as price consultants and deal makers. Specifically, this order was ruled by approximate formulae, among which Halley's formula of 1706 reigned supreme. We argue that approximate formulae were good enough because landed property was a decentralized market: Parties received a general direction, after which they could be left to haggle their way to agreement. Algorithmic methods, by contrast, served to cultivate the platonic ideal that, if push came to shove, things could be calculated exactly, and so they fulfilled a vital function, even as they were rarely mobilized in practice.

Coffee House Computing

In the English world, the rise of the financial intermediation function of algebraists occurred through their intervention in the pricing of landed property, then a predominant store of wealth. The real estate market was standardized in a series of rental contracts known as leasehold, copyhold, and freehold, which committed the tenant (lessee) to make annual payments to the landlord (lessor) for given time periods.[74] Against this backdrop, land ownership could be formalized as an annuity contract, the corollary being that the sorting of value hinged on the ability to actualize alternative contracts and extract their respective returns. From there arose a demand for that peculiar knowledge which enabled resolution of differences and permitted parties to come to an understanding. As Bellhouse puts it, "given this system of land

[74] Copyholds and leaseholds had durations that could be either fixed or dependent on such things as the life of the lessee. The default position of freeholds, by contrast, was to be perpetuities, essentially a form of modern ownership.

tenure, it was in the landlord's interest to be knowledgeable in the calculation of interest and fixed-term annuities."[75]

A further consequence of this was that, given a *capacity* to sort this kind of problem, it was advantageous to structure *other* contracts along similar lines. A central tenet of Bellhouse's book is the formal identity between the pricing of life- or term-based real estate rights and the pricing of life annuities, so that, to recover the mathematical origins of life insurance, we must look beyond life insurance and toward the broader financial polity. But this manifested a much broader logic. As was already acknowledged by contemporary observers, mastery of the annuity problem would serve to address about anything. This was how Francis Baily cast the matter in the early nineteenth century: "The importance of [annuity pricing], at the present day, cannot be doubted; *since the greater part of the property of this kingdom is, in one shape or another, connected with this science*."[76]

Next, the market for leases was not centralized. Rather, it was an over-the-counter market, whereby parties entered in bilateral contact and came to an understanding. Transactional fluidity hinged on the ability to create this understanding. Mathematics thus became the "place" where value was sorted, enabling heterogenous leases/annuities to be converted into notional equivalents and in this manner, to one another. Thus, responding to what may be described as the pressure of the market, the mathematicians of wealth developed a set of tools to assist traders with financial calculations. By nature, these tools tended to propagate a universal understanding of value, increasing the likelihood that two parties meeting in any "social platform" (a coffee house, a salon) would likely share similar principles.

In practice, it fell upon a group of English mathematicians to help create the foundations of the required trade enhancing knowledge system.[77] The leading figures of the group were Isaac Newton, Joseph Raphson, Edmond Halley, Abraham De Moivre, and Thomas Simpson. Newton, Halley, and Simpson contributed to the development of efficient solutions to the annu-

[75] Bellhouse, *De Moivre*, 155. An example of the kind of computations that operators contemplated is provided by Baily. As he discusses the interest rate used in practice, he declares: "In freehold estates this is always taken into the account, and the true values are easily deduced accordingly [because the interest rate is automatically deducted from the price of the perpetual lease and the rent]: so, by the help of the following tables, [the values] may be as readily ascertained in the purchase of any leasehold estate [because once the interest rate is known then the price can be deducted from knowledge of the maturity]" (*Purchasing and Renewing*, 1802, 6).

[76] Baily, *Doctrine of Life-Annuities*, iii–viii, our italics.

[77] Bellhouse, *Leases for Lives*, 8–24.

ity problem. Newton and Raphson developed a method, which would be improved by Simpson in 1745, then by Maseres in 1804 to design powerful update functions. Raphson was a protégé of Halley, who lobbied for Raphson's admission to the Royal Society.[78] De Moivre's contribution was the generalization of the annuity problem to increasingly complex contingent events.[79]

Thus, the mathematicians of wealth provided the mainframe for the market for leases' decentralized exchange. All that the mathematicians of wealth needed to do was provide a ballpark. For instance, an advertisement De Moivre inserted in 1739 in the London press showed that he would—from his haunt at Slaughter's Coffee-house—examine pricing problems submitted to him.[80] As the ad explained, the consultation would enable the inquirer to hear from De Moivre "what the Rate of Interest is, which is agreed between the contracting parties." The language is remarkable, for the impersonality it projects. In some cases, mathematicians would go the extra mile and directly assist pricing.[81] De Moivre's business drew on the prestige of his successful mathematical publications, which in turn enabled him to act as deal maker for transactions among persons of property. Being himself a Huguenot, a very active financial milieu, he was well connected to the monied interest.[82]

The construction of the relevant knowledge was anything but impersonal, however. It rested on the construction and maintenance of social networks, operating in practice as trading platforms. Studies of early modern scientists indirectly evoke this, as reflected by the ubiquitous use of the term "circle." Contemporary scientists operated inside circles of sociability and conversely, his *circle* of individuals orbited a scientist. When it came to numerical techniques mobilized to sort out the problem of value, we argue that the circle came to define an economic space. This space was governed by the mathematician, who served as arbiter of last resort, delivering the impersonal *judgment* of mathematics. The mathematician was an expert in fairness, equity and good conscience.

The study by Bellhouse et al. of the subscribers sponsoring Abraham De Moivre's *Miscellanea Analytica* (1730), identifies three groups appearing

[78] Halley's recommendation praised his method of "Solving all sorts of Aequations." See Kollerstrom, "Thomas Simpson," 348.

[79] De Moivre, *Annuities upon Lives*, 5.

[80] Bellhouse, *Leases for Lives*, 81.

[81] Bellhouse, *De Moivre*; Bellhouse, *Leases for Lives*.

[82] Contemporary descriptions of Exchange Alley indeed never omitted a picturesque reference to French refugees involved in financial business.

in roughly equal numbers. There were landed aristocrats, Whig politicians, and members of the Royal Society (and university fellows).[83] The authors speak of a "knowledge community," but it should be borne in mind that an *economic* interest—the need to ensure liquid trading of assets—provided the glue that kept such communities together. The landed aristocrats wanted a pricing infrastructure. The commitment of Whig politicians to the development of a funding system for British debt helped them realize that what would happen gradually would be a progressive swapping of land leases for financial assets.[84] As for the scientists, they found resources and status to advance their own interests.[85] So, although it is tempting to say that the prestige commanded by scientists facilitated their successful intervention in addressing questions of value, it is also possible that the causality worked the other way around—that a not insignificant portion of their prestige as scientists came from the fact that they provided vital help as financial capitalism spread its wings.

Related to the above, an important aspect of the manner in which value was governed in the age of the mathematicians of wealth concerns the distribution of formulae. Trading subjects had to see eye to eye, which meant, as already said, a shared understanding. Against this backdrop, the distribution of computing programs played an important role as it supported value making. This involved, first, teaching. Teaching operated as a kind of formatting. It prepared asset holders for the marketplace. As a result, interest and annuities were part of the curriculum. Bellhouse explains that De Moivre built his consulting clientele through his work as a tutor for the children of the elite.[86]

Daughters were not left out. In 1751, the popular *Ladies' Diary*, a mathematical almanac in whose publication Thomas Simpson was involved and which boasted around 7,000 readers (not all of whom ladies), posed a problem that amounted to computing the yield on a five-year annuity of £25 per year, for which one had paid £73 (see box 2). The reader was prompted "to find his gain percent according to the allowance of compound interest." The solution led respondents to mobilize their command of Halley's mathematics—specifically, his approximate formula. Discussing

[83] Bellhouse et al., "De Moivre's Knowledge Community"; De Moivre, *Miscellanea Analytica*.

[84] Dickson, *Financial Revolution*.

[85] Flandreau, *Anthropologists*, 1–16, makes a case for studying the "financial interests" of science.

[86] Bellhouse discusses "a number of students from landed families, some of whose associations with De Moivre go back to the late seventeenth and early eighteenth centuries" (*De Moivre*, 156).

Box 2 Lady Computers

In the *Ladies' Diary: Or, Woman's Almanack* of 1751, p. 40, "Χρονονομονπυβλικ" (no man public?) posed the following problem (N° 345): "If a bookseller buys a copy for 21 £. pays 21 £. for paper, 21 £. for printing 500 impressions, and 10 £. for advertisements and other contingent expenses, amounting in all to 73 £. and sells 100 books yearly of the *History*, at 5 *s.* each, what is his gain per cent. allowing compound interest, for the time he lies out of his money?" As was the tradition, solutions were offered in the next year's edition of the *Ladies' Diary*, in this case by "Upnorensis" (1752, 39–42).

IX. QUESTION 345 *anſwer'd by* Upnorenſis.

THE Bookſeller buys an Annuity of 25 *l.* a Year, to continue five Years, for 73 *l.* ready Money—To find his Gain *per Cent.* according to the Allowance of Compound Intereſt?

Let $\left\{\begin{array}{l} a \\ r \\ t \\ z \end{array}\right\}$ ſignify $\left\{\begin{array}{l} \text{the Annuity.} \\ 1\ l. \text{ and its Intereſt for one Time or Year.} \\ \text{the Number of Times the Annuity is to be paid.} \\ \text{the whole Amount of the Annuity's preſent Worth.} \end{array}\right.$

C 4 Say

Upnorensis noted that this problem could be formulated mathematically as a finite geometric series. Having provided the general formula, he went on using Halley's direct formula to solve for *r*.

The aforeſaid Equation reduces to r^{t+1} *min.* $\frac{a+z}{z} r^{t} + \frac{a}{z} = 0$; in which, according to Dr. *Halley*, if t the Number of Years be great, 40 or upwards, and the Rate of Intereſt be high, $1 + \frac{a}{z}$ will be nearly, or more accurately $\frac{z+a}{z} - \frac{z^{t}}{z+a} \times \frac{a}{z}$, the Value of r, when $\frac{a}{r^{t} \times \overline{r-1}}$ will be exceedingly near the Value of the Reverſion, which, if it be call'd x, then $1 + \frac{a}{z+x}$ will approach the Value of r ſufficiently. *See Dr.* Halley's *Method, at p.* 33 *and* 34 *Appendix to* Sherwin's *Logarithms.* But, t the Years being *ſmall*, this Rule fails; in which Caſe, if $\left(\frac{at}{z}\right)^{\frac{1}{\frac{1}{2} \times \overline{t+1}}} - 1 = y$, and $\frac{6}{t-1} = b$, then

$1+b-\sqrt{bb-2by}$ is ſufficiently equal to *r*, and will ſtill be nearer the Truth, as *t* the Years be of the *ſmaller* Value; the ſmall Error being always in *Exceſs*: *viz.* $r = 1 + b - \sqrt{b \times \overline{b - 2 \times \frac{at}{2}}^{\frac{1}{\frac{1}{2}\times\overline{t+1}}} - 1}$. And, putting *Capitals* for the Logarithms of Quantities denoted by ſmall Letters, then $\frac{A+T-Z}{\frac{1}{2}\times\overline{t+1}} - n = D$, and $\frac{B + L.\overline{b-2\times d-1}}{2} = E$, Th. $r = 1 + b - e$; and conſ. $r - 1 = b - e = ,21094$, &c. the *Rate* of Intereſt of 1 *l.* *per* Year, and 21 *l.* 1 *s.* 10½ *d.* for 100 *l.* a Year, Bookſeller's Profit. Q. E. I.

As can be seen, Upnorensis explained Halley had proposed two methods, one for $n \geq 40$ and one for $n < 40$. Since the annuity in the problem matured in five years, the second applied. Upnorensis then proposed Jones's version of Halley's direct formula presented, for example, in Gardiner's *Tables of Logarithms*, p. 8. Remarkable fact: Upnorensis gives the "solution" from "Halley's method" as 21.094%, yet application of Halley's method gives 21.123%. In fact, 21.094% is the *exact* solution of the problem to three decimals.

the solution in the next issue of the *Diary*, "Upnorensis" did not omit to praise Halley, "who scarcely had his Equal, for Judgment, Art, and Method in the Things."[87]

The other aspect of distribution was propaganda. Propagating the mathematics of wealth served to homogenize the "software" that equipped individual computers. "Distributors" emerged who published treatises that provided user-friendly versions of the highbrow "theorems," thus operationalizing the program. As indicated, the answer the *Ladies' Diary* proposed to the bookseller's problem mobilized Halley's formula. The solution explicitly referred to Halley's chapter in Henry Sherwin's *Mathematical Tables*, where the formula had first appeared. But it also gave the variant of Halley's for-

[87] Upnorensis, "Question 345." On the *Diary*, see Perl, "*Ladies' Diary*." Bassett, "Goat in the City" suggests "Upnorensis" might have been an alias of Thomas Simpson (editor of the "Diary" between 1751 and 1860) but concludes that mathematician Robert Heath was more likely.

mula developed by William Jones, which had appeared in William Gardiner's *Tables of Logarithms*, as well as in James Dodson's *Anti-Logarithmic Canon*.[88] John Robertson explained in 1770 how these variants spread. Jones (who belonged to Halley's circle) had a collection of compound interest rate problems engraved in copper plates. This early print-on-demand system facilitated distribution.[89]

Local Knowledge Monopolies

Against this backdrop, we grasp how the early modern government of asset pricing was structured. We should picture value being sorted within individual communities, loosely coupled to one another yet generally inspired. Because each community had its own rules, there could be differences across the broader polity. This was tolerable and tolerated. One group could use Halley's approximation while another could use Simpson's. At the end of the day, the formulae were only making market possible. Since the operations consisted in the trading of leases or leases-assimilated contracts, to be conducted over the counter, significant margins for negotiation were involved, which means that approximations were good enough.[90]

The point is, the notion that prices existed was more important than the exact way they were calculated. They could be, and were, calculated in many ways. As a result, we might say that, given "objective" prices, there were in each bargain "winners" and "losers." But this did not matter: What mattered was to entertain the idea of truthfulness out there. Then the circulation of wealth was facilitated. It was fine from this vantage point that there existed side by side "Halleyan" tribes and perhaps elsewhere (or later) "Simpso-

88 Upnorensis, "Question 345," 41. Gardiner, *Tables of Logarithms*, 8; Dodson, *Anti-Logarithmic Canon*, 53. These were nothing but Halley's formula, except that in Jones's treatment, it was recast to enable calculation through extensive use of logarithmic tables. For the record, Jones's career as a vulgarizer had started with the publication in 1706 of his *Synopsis Palmariorum Matheseos*, "design'd for the benefit and adapted to the capacities of beginners" [Jones, *Synopsis*, cover]. This was the same year Halley's direct formula had been released and, at this point, Jones was still directing readers to Dary's approximate formula.

89 Robertson, "Letter to James West," 508; Baily, *Doctrine of Interest*, viii, drawing on Robertson.

90 Baily observes, "On this point, no doubt, difficulties will sometimes arise; for, the value of an estate, depending very often on some real or supposed advantages, or on some local or personal recommendations, will, in many instances, occasion a difference of opinion" (*Purchasing and Renewing*, 1802, 7). He carries on to explain that of course the method of estimating the value of the "rack-rent" (i.e., the lessee's rent) of estates is a different art altogether.

nian" ones. Think of it as a form of religious toleration: Each adored a version of God.

What was essential was thus that there existed a notion of truthfulness. Independently of the way actual solutions were derived, the existence of the mathematics of wealth provided blanket reassurance that price truthfulness wasn't an illusion. It provided the promise that (as De Moivre's advertisements put it) the agreeable interest rate would be revealed. The consequence of this was the detachment that prevailed between calculation problems and the problem of calculation. To put it differently, while value *measured* was always local and bilateral—a price to which the contracting parties consented—value *conceived* was anchored in a universalist understanding that did not need to be calculated to exist. But then, of course, because of this, computing and computers were placed at the epicenter of financial capitalism.

This was consistent with—and even conducive to—a tendency for some formulae to dominate. Virality trumped exactness. We just caught a glimpse of the success enjoyed by Halley's direct formula, which was relayed by various authors. Its widely attested popularity was owed to the centrality its author enjoyed in English science, that is, to the size of Halley's own circle, which encompassed many mathematicians he was actively supporting.[91] The other related aspect was that Halley's formula (1706) benefited a head start.[92] Halley's approximate formula continued its rule, even after Simpson's came out, and even as the precision of the latter was superior. The system as it was, resting on convention bolstered by the *possibility* of absolute precision, accommodated very well approximation, especially if it was due to a man who scarcely had his equal for judgment, art, and method. In 1808, Francis Baily would still describe a situation where Halley's "theorems," duly "transcribed and formed into rules by various authors," provided in practice its foundations of financial capitalism's pricing system.[93]

[91] And so, for instance, if we go back to the example from the *Ladies' Diary*, while Jones was directly connected to Halley, Dodson was a student of De Moivre, who belonged to Halley's network.

[92] In addition to Gardiner and Dodson, already mentioned, the promoters of Halley's formula included Robertson, "Letter to James West," 514; Wilkie, *Theory of Interest*, 76; Callet, *Tables Portatives*, 71.

[93] Baily, *Doctrine of Interest*, vii.

Hackers?

The above description naturally leads us to "predict" the possible emergence of charlatans who would try to peddle their wares to an unsuspecting public. An illustration is provided by the case of James Hardy. Little is known about this calculator except that in 1753, he published a *Complete System of Interest and Annuity* (1753), a bizarre treatise that attacked compound interest, advocating simple interest. Based on this, Hardy built an alternative "system" that included calculation of present value, extraction of the interest rate solution, and tabulation. Hardy praised his "system" "as exact, as any that have been hitherto made Public," "more easy and expeditious," "built on a more rational plan," and "more conformant to reason."[94]

As later authors would underline, simple interest, which was by no means Hardy's invention, was hardly more rational and was only useful as a convenient approximation for short maturities.[95] Nor were the resulting formulae in Hardy's "system" in any way exceptionally powerful. As it turns out, his (approximate) formulae for interest rate determination were not even better (in fact worse) than the material already published on this arcane subject.[96] In the nineteenth century, actuary Peter Hardy (no connection

[94] Hardy, *Complete System*, unnumbered last page of the preface, 3.

[95] Unlike compound interest, it did not allow interest on interest. Simpson, specifically referring to James Hardy, states: "the valuation of annuities according to simple interest [is] a matter of more speculation than real use, it being not only customary but more equitable to allow compound interest" (*Treatise of Algebra*, 1755, 235). De Morgan argues that "simple interest, as applied to annuities, is really a fiction not more worthy of attention than would be the transactions of a man who, having a number of sovereigns in his hand, and others coming in, should invent a rule for separating some from the rest, and locking them up in a cash-box instead of investing them" ("Calculation of Life Contingencies," 143).

[96] James Hardy's formula to calculate the present value of an annuity at simple interest (*Complete System*, 7) was less precise than that which Dary had devised on the same issue in 1677 (*Interest Epitomized*, 28). Consider a unit annuity discounted at 10%, maturing in 10 years. At simple interest its present value at two decimals is 6.69. James Hardy's formula gives here 5.92 (error of 0.77), while Dary's formula gives 7.25 (error of 0.56). Note that the series representing the present value of an annuity at simple interest $(1/(1+i)+1/(1+2i)+ \dots +1/(1+ni))$ does not admit any closed-form general expression. Calculation of an approximate solution with any reasonable degree of precision for such a case does rest on the use of the Euler-Maclaurin summation formula, which essentially consists in approximating the sum of a function by the integral of that function. Such a method was first applied in actuarial science in the 1860s, at the initiative of W. S. B. Woolhouse ("Improved Theory of Annuities"), to calculate the value of complex life contingent contracts. As far as we can tell, De Morgan was the first to apply the Euler-Maclaurin formula to approximate the present value of an annuity at simple interest ("Calculation of Life Contingencies," 143–44). Equipped with that tool, and discussing the numerical example presented above, De Morgan found the "true present value" at five decimals' precision. While comparing the various

with James) would describe James Hardy as an "obscure writer" and imply he might actually have lifted his formula somewhere.[97] Given that Hardy's approach approximated an atypical method (simple interest) with a flawed formula, it is easy to understand why he was treated as an outcast.[98]

A likely interpretation of what James Hardy was doing is that he was attempting to carve for himself a share in the rapidly expanding market for price consulting. Edwin Kopf describes a mid-eighteenth-century annuity treatise frenzy, coinciding with accelerated financialization of the British economy. The result was surging demand for guidance in financial calculus.[99] Hardy perceived that the success of the consulting function of the mathematics of wealth was rooted in prestige procured by theoretical and practical achievements and reverse-engineered the approach followed by Halley, De Moivre and the others. James Hardy understood, too, the importance of patronage. But since, unlike Halley, De Moivre, and the others, he did not have a grandee in his pocket, he sought to aggrandize himself by dedicating his treatise to then Prime Minister and Chancellor of the Exchequer Henry Pelham.[100]

Hardy's tactics illuminate the nature of the early modern numerical order. The decentralized character of numerical governance enabled the rise of "fly-by-night" computers. While a priori, this entailed the risk of compromising the financial system, the case of James Hardy may be read as underscoring the limits of the threat. After all Hardy contributed by adding his own work force and mental energy (albeit in a distorted way). As already suggested, if those parties who heeded his advice encountered contrasted

formulae of present value at simple interest, King remarked that James Hardy and Dary's formulae (designated as formulae [5] and [3], respectively, as King did not name these two authors) produced results which were "very wide of the mark" (*Theory of Finance*, 1882, 196–97). On this issue more generally, see Sutton, *Institute of Actuaries' Text-Book*, 24–25, 141–44.

[97] Peter Hardy states, "at least this author speaks of the rule [in question] as being his own invention" (*Doctrine of Simple and Compound Interest*, 13).

[98] For instance, for a range of theoretical annuities all returning 5%, whose maturities varied between 10 and 50 years, the approximations produced by James Hardy's formula ranged from 4.5% to 4.7%, that is, at best 30 basis points away from the exact yield. In contrast, the approximation produced by Halley's direct formula ranged between 5.0006% and 5.087%.

[99] Kopf lists eight contributions to the mathematics of finance between 1739 and 1763 ("Early History," 257).

[100] Prestige by association was a widespread commercial tactic. Baily tells the story of the *Tables for Renewing and Purchasing of the Leases*, published anonymously in 1685, with an endorsement by Isaac Newton. After the latter's death, the "publishers artfully prefixed to the book, in large capital letters, the alluring title of Sir Isaac Newton's Tables, under which name it has erroneously gone ever since" (*Purchasing and Renewing*, 1802, iv).

outcomes, *they nonetheless were compelled to participate in the circulation of capital*. A further question is how extended James Hardy's "circle" was. For one thing, his book is found in the library of George Washington, who was twenty-four years old when the *Complete System* came out. It is not known if the future US president did anything with it.[101]

Algorithms as Kabbalah

The dominance of approximations did not rest on cognitive limitations that would have led contemporaries to satisfy themselves with whatever level of precision they could achieve, as some modern authors have been inclined to speculate.[102] Rather, it was the by-product of a knowledge order that was governed in such a way that it tolerated local departures—local variations of the universal dogma—precisely because the dogma was so firmly entrenched. In fact, albeit this may escape superficial examination, this knowledge order rested on algorithms, the apostles of extreme precision. Algorithms played a fundamental role in underpinning this regime, because their mere existence ensured that belief in exact solutions was warranted. This vouching function arose because to assess the "validity" of an approximation, calculation of the "true" value was required.

From there, two conclusions follow: First, algorithms were an essential aspect of the early modern computing order. Even when they were not used in practice, they provided reassurance that a method existed out there to ascertain the true value with arbitrary precision. In other words, truth existed. In the language of Edmond Halley, algorithms enabled to "prosecute [the] enquiry as far as you please."[103]. Second, this situation contributed to placing the mathematicians of wealth at the center of the machinery that adjudicated value. Indeed, the fact that they commanded the dark arts of algorithms turned them into the natural authorities of this adjudication process, enabling them to act as ultimate gatekeepers of early modern financial mathematics. They were tasked with assessing sound formulae and detecting fraudulent ones, which in the end enabled them to limit the threat

[101] See catalog of George Washington's Presidential Library, Mount Vernon, VA, at https://www.mountvernon.org/library/special-collections-archives/books-owned-by-washington.

[102] Mauro, Sussman, and Yafeh, *Emerging Markets*, 41–42, suggesting that approximations were good enough for our forerunners.

[103] Halley, "Compound Interest," 35.

posed by the latter. Defeating notions of "open science," algorithms thus lived an underground life.[104]

A characteristic instance of the secret role played by algorithms in the theory of wealth is provided by Thomas Simpson's *Treatise of Algebra* which, as we saw, contained two approximate formulae for the annuity problem (sec. 16, 212–13) but *also* contained a section where the author gave a general method for the exact resolution of high-order polynomial equations (sec. 12, 125–64). Simpson referred to it as the "converging series method," adding it was common knowledge among algebraists of his time.[105] Simpson did not mention its relevance to the case of annuities in this section, but, of course, he had to be aware of it. Indeed, in the section devoted to the annuity, he declared that it could be sorted with "the Method of Adfected Equations," that is, with the general method for higher-order polynomials presented in section 12.[106]

Accordingly, although Simpson did not give the formula, we are allowed to derive it using the cooking directions he left. As said, this leads to a striking discovery: We land on the update function "discovered" by Maseres in 1804, which must therefore have been known and used by Simpson sixty

[104] This emerges clearly when we compare popular treatises and "scientific" ones. The former only focused on providing practical guidance and usable formulae, which led them to ignore algorithms. The latter by contrast did engage the subject. Indeed, quite apart from questions of performance, algorithms provided proof that numerical truthfulness existed. See for instance, the presentation of Halley in Gardiner's *Tables of Logarithms* is silent on his interest in algorithms.

[105] Simpson, *Treatise of Algebra*, 1745, 147. This "converging series method" harkened back to Newton and Raphson, who had themselves drawn from François Viète (1540–1603) and William Oughtred (1574–1660). As far as one can tell, Halley himself appears to have missed out on the method, although as we saw, he had praised Raphson's approach. As Maseres put it, had Halley applied "the original reasonings and principles upon which Mr. Raphson's method is grounded," he would have derived a significantly more powerful update function than the one he came out with ("Some Difficult Passages," 297). Kollerstrom argues that the so-called Newton-Raphson method was really developed by Simpson ("Thomas Simpson"). Newton and Raphson's algorithms were variants that anticipated the general method formalized by Simpson. It exploited the local properties of polynomials to generate a closed-form update function $\varphi(i)$ for trial values of the unknown solution i. To begin, the update function was written as the sum of the trial value assumed to be close to the true value and of an error term x, which would be small as a result, or $\varphi(i) = i + x$. This expression could be substituted into the equation to be solved. In the case of a polynomial, the error term x being small, its powers higher than one could be ignored and the resulting linear equation solved for the error term x as a function of i. This generated the algorithm's update function, which could be applied iteratively enabling, as Simpson explained, to return "the value sought, *with little trouble, to a very great degree of exactness*" (*Treatise of Algebra*, 1745, 147). See also Cajori, "History of the Arithmetical Methods"; Ypma, "Historical Development."

[106] Simpson, *Treatise of Algebra*, 1745, 213.

years earlier.[107] In fact, by examining the annuity problems discussed by Simpson, one becomes quite certain that he used the formula in question. On pages 212–13 of the *Treatise*, for instance, he gives an annuity problem for which the true value can be calculated, as Simpson indicates, with four decimals' precision, as 4.2775%. How did he find out that this was the true rate at that degree of precision? Using Simpson's shadow algorithm and Smart's table for starting values, we arrive at this number after three iterations only.[108] It is unlikely that Simpson would have used any other algorithms extant, given that they would have involved many more steps.[109] Simpson describes his own experience with this specific calculus as "somewhat tedious," which sounds like an adequate description of the nontrivial travails required from application of the algorithm. We note, finally, that Simpson urged readers to stick to approximations.[110] This makes our point that in the Republic of Numbers, the governance of value was *informed* by approximate formulae yet *backstopped* by algorithms.

The point is that even when they were ostensibly not used, algorithms stood guard. In the hands of crafted individuals, they did uphold and detect. They remained within the specialist sphere, hidden, secret, an instrument to govern approximations and protect the mathematical intermediaries' informational rents. Evidence of this occult role of mathematics speaks to previous conversations regarding the role the mathematics of wealth played in early financial development. There is no contradiction between, on the one hand, the multiplication of mathematical treatises relevant to finance including life insurance, and, on the other hand, the fact that mathematicians were *not* involved in every plan. By vouching for the broad feasibility of certain schemes and elucidating the price that was given when specifically asked, early modern financial mathematics provided a frame of thought. The understanding which they promoted, that to each well-defined problem corresponded a well-defined concept of value, was vital in that it enabled making the market.

That was not the same as directly designing the products. In fact, given the reputational risks involved, it is not clear why the mathematicians of

[107] This explains why in table 1 we refer to the "Simpson-Maseres" formula.

[108] We applied Simpson's converging series method formulated on page 149 to the annuity problem stated on page 213 in his *Treatise of Algebra*, 1745.

[109] At that date, there were only two other update functions. Beginning with the same starting values, Dary's formula would have required 37 iterations before converging, Halley's 118. Another possibility is that Simpson used a table along with an educated guess, such as the false position.

[110] Simpson, *Treatise of Algebra*, 1745, 211, 213.

wealth should have ever wanted to stand at the forefront. Actual business occasions were the result of myriad experimentations, adjustments, and departures from the template, the work of individuals described as "Projectors" by Postlethwayt's *Dictionary of Trade and Commerce*, those who "contrive" and "scheme." As he states, "and from these sorts of men, it is easy to trace the origins of BANKS, STOCKS, STOCK-JOBBING, ASSURANCES, FRIENDLY SOCIETIES, LOTTERIES, and the like."[111] As far as the interests of the mathematicians of wealth were concerned, such projects were expected to stand on their own legs—or not at all. The mathematicians "patented" a method but guarded themselves against vouching for individual commercial experiments. While we find them occasionally in hands-on roles (as in the case of De Moivre's consulting activities), a more fitting role was to intervene in pricing panics, that is, when the situation was ripe for computers to reassert their moral rights over calculus.[112]

An illustration is P. G. M. Dickson's discussion of Archibald Hutcheson, mobilizing the mathematics of wealth in 1720 to criticize the high valuation of South Sea Company shares. Hutcheson contended there was a bubble because current prices implied implausible future dividends. In modern language, we'd say he was "testing" for a bubble.[113] Importantly, Hutcheson's evidence consisted in guiding readers through an annuity table so that they could see it for themselves.[114] Another example is provided by Bellhouse's study of responses to the life annuity "bubble" of the 1760s and 1770s. As he shows, a group of calculators went after poorly priced policies.[115] On each occasion, computers used persuasion strategies that assumed readers had *some* command of relevant mathematical concepts (present value, annuity problem). This speaks of a prior "formatting" of the public and, what is more, gives suggestive evidence of what may be described as a primitive accumulation of numerical capital without which capitalism could never develop.

[111] Postlethwayt, "Projector," 553.

[112] The role of land lease consultants in putting the insurance business on actuarially sound footing is made by Bellhouse, *Leases for Lives*: on Dodson and the Equitable, see 111–17, on Price and Morgan, 198–205.

[113] Hutcheson, *Calculations Relating to the Proposals*, 2–6. On Hutcheson's calculations, see Dickson, *Financial Revolution*, 101–2.

[114] Hutcheson invites his reader to "compute for himself, from a Table of Compound-Interest," the dividend the South Sea Company ought to pay for the securities to be worth the price ("Seasonable Considerations," 30).

[115] Bellhouse, *Leases for Lives*, chap. 8, "The Annuity Bubble of the 1760s and 1770s."

A Theory of Francis Baily (1800–1848)

We suggest the second computing order was ushered in by Francis Baily's 1808 treatise, *Doctrine of Interest and Annuities*. Breaking with existing habits, Baily was speaking in favor of an exacting level of numerical accuracy, "sixth place of decimals" or a four-digit precision.[116] This was in fact a rallying call for algorithms, since they alone could deliver such precision. What led Baily to advocate this program? We answer that it had a lot to do with the emergence of the London Stock Exchange as the primary site for trading assets. Because the London Stock Exchange was a centralized market, more closely calculated prices became necessary, lest scandalous price divergences would come into the open. One particular group—to which Baily belonged—had reputational skin in this game. Being buyers and sellers of securities, "jobbers" such as Baily held inventories of stocks and provided liquidity by continuously quoting buy (bid) and sell (offer) prices. They faced the risk of being upset, if numerical heresies took hold. It only makes sense that a program for putting calculus on a uniform basis should have emerged from their ranks.

Histories of Francis Baily

Such is not the way the story of Baily's *Doctrine* has been told before. As we already have emphasized, Francis Baily's *Doctrine of Interest*, which provided in 1808 a state-of-the-art survey of financial calculus, is generally recognized as towering over subsequent actuarial literature. Historian William J. Ashworth has related Baily's 1808 project to his *subsequent* interventions in astronomy. Along with several other financiers, Baily would be involved in 1820 in the creation of the Astronomical Society.[117] Ashworth, who is not interested in the history of financial calculus, asks why a "group of professionally oriented people such as bankers, insurance men, merchants, lawyers and accountants" got involved in astronomy.[118] His answer is that "business astronomers" invested in astronomy with the purpose of elevating their own standing (which Ashworth appears to believe needed redemption). This is how Ashworth cast the matter: "This group of men typically founded

[116] Baily, *Doctrine of Interest*, 121.

[117] Ashworth, "Calculating Eye," 416.

[118] Ashworth asks what led "business astronomers" to try and recycle the "commercial values of probity, punctuality and prudence" as the astronomers' "values of vigilance, patience, precision and calculation" ("Calculating Eye," 441).

[learned] societies and were interested in projecting a cultured philosophical image."[119]

Joel Mokyr (drawing on Ashworth) has provided yet another theory of Francis Baily. This time, Baily is branded a "typical product of the late British Enlightenment, expressing his horror of American slavery after a tour of the United States as a young man."[120] This hatred of slavery, speaking of a commitment to Enlightenment values, would explain in Mokyr's view why Baily should have been inspired to "place commercial computation on a scientific basis." But Baily's contribution was not to provide scientific foundations of financial calculus. As we have seen, these had been laid out long ago by figureheads of the Enlightenment such as Halley. What is more, extreme precision was readily attainable with available techniques, as demonstrated by the availability, at least among skilled persons, of Simpson's secret algorithm. The question of what led Baily to suddenly *advocate* precision remains open.

We suggest that the central character is not Baily, but the London Stock Exchange, of whom Baily was a prominent member. As for the advocacy of algorithms, it resulted from the centralized character of security trading at the London Stock Exchange. Centralized trading opened a clash between competing pricing models and created concerns for market makers such as Francis Baily. As a result, stockjobbers who as said stood ready to buy and sell securities at posted prices, supporting a fast turnover, found it was in their interest to promote uniform understandings of value. Stockjobbers being the elite of the market, they controlled its governing committee, conferring them a unique ability to propagate norms. As a result, a new breed of mathematicians emerged at the center of financial mathematics, replacing the mathematicians of wealth.[121]

In other words, we interpret Baily's support for enhanced precision as driven by a market imperative. Baily had started his career in 1799 by joining James Penfold, a senior stockjobber.[122] That Baily demonstrated a

[119] Ashworth, "Calculating Eye," 416. He continues: "Accordingly, the business astronomers set out to develop a trustworthy approach to the accumulation of intellectual and financial capital."

[120] Mokyr, *Enlightened Economy*, 231.

[121] Their role differed from that of the brokers, who acted on behalf of clients for a commission.

[122] Charles Hammond and Son Papers, London Metropolitan Archives, CLC/B/044/MS21594. As verifications performed in the London Stock Exchange Membership Applications lists 1802–1812 show, Baily's name appears on every membership list following the transformation of the Stock Exchange Coffee House into a voluntary association in 1801. Membership lists are available at ancestry.com, London Stock Exchange Membership Applications, 1802–1924.

concomitant interest in the mathematics of the annuity is demonstrated by the fact that his first treatise was published in 1802.[123] There is compelling evidence that Baily's numerical work had institutional significance. Indeed, he was a champion of the market's interest, and of the interests of stockjobbers especially.[124] In 1808 he published the *Doctrine of Interest* and in 1810 he was elected to the stock exchange's governing committee.[125] In 1811, he became the committee's vice chairman; in 1812 he was part of a select committee appointed to draft the rules and regulations of the London Stock Exchange.[126] Baily's scientific interventions are to be understood in the context of his functions at the stock exchange, rather than as a result of his personal beliefs or attitude toward slavery.

Pricing Scandals

Looking at the London Stock Exchange's price list provides evidence of the existence of a computing ecology centered on the annuity. Indeed, it contained a profusion of annuities of various kinds.[127] Both the chartered companies (East India Company, Bank of England, South Sea Company) and the British government, which was emerging in the 1790s as the principal client of the London Stock Exchange, were avid users of this instrument. Consider for instance Fairman's *Stocks Examined and Compared* (1798). As Fairman demonstrates, the *entire* fixed-income segment of the stock market (by far the largest portion of the market itself) consisted of annuities.[128] By relying on the already existing template and thus reinforcing its pull, the

[123] Baily, *Purchasing and Renewing*, 1802.

[124] For instance, in 1806, Baily found himself litigating the Corporation of London to secure a special status for jobbers. See Baily, *Rights of the Stock-Brokers*.

[125] Armstrong's *Book of the Stock Exchange* has a useful list of chairmen and vice chairmen (378).

[126] Stock Exchange Committee, *Rules and Regulations*. Interestingly, the one example available online is marked as "one of Francis Baily's publications" and was presented to the Astronomical Society by astronomer S. M. Drach.

[127] For instance, the British Deferred Annuities of 1802 started paying in 1808.

[128] Fairman, *Stocks Examined*. The bulk (93%) of the funded national debt was in perpetual annuities (or Consols, short for "consolidated annuity"). Besides these perpetual annuities, the government had issued several fixed-term annuities over the course of the eighteenth century. The "Long Annuity" issued in 1761 matured in 1860. Additional issues of the same product continued for half a century. Ten-year "Short" annuities were issued in 1777; the "Irish Terminable Annuity," issued in 1794, had a maturity of 15 years. The "Imperial Terminable Annuities" issued by Austria in 1795 (under British government guarantee) had a maturity of 25 years. Taken together, fixed-term annuities accounted for 6% of the national debt.

rise of the London Stock Exchange perpetuated and amplified the rule of the annuity problem as *the* quintessential quandary of value, as Baily would emphasize in the *Doctrine*.[129]

At the same time, irruption of the London Stock Exchange as the new, dedicated center for asset trading introduced novel elements. Product diversity *within* the universe of annuities encouraged investors to perform arbitrage. In fact, the peculiar way in which Prime Minister William Pitt's war loans of the 1790s were structured both assumed and encouraged such knowledge. Each individual loan consisted of a portfolio of several securities (typically three). For every £100 advanced, bidders would receive fixed amounts of the securities in the portfolio but one, in which the auction was conducted. Large syndicates vied with one another to win the contract the winner being the one prepared to receive the smallest amount of this security. In the 1790s, bidding was conducted in Long Annuities while the other securities were Consols (4% and 3%). In other words, the chosen technique required bidders to think hard on the relative price of government securities.[130]

Of course, the syndicates of bankers and brokers who lined up in such auctions had the means to secure relevant computing talent. However, the syndicates did not intend to keep the securities forever. This raises the question of which model informed final investors. Several authors peddled tables that were designed to help users answer arbitrage questions "at a glance." An example is provided in Thomas Mortimer's popular do-it-yourself guide to stock exchange investing.[131] It gave a so-called "Table of Equation" built on the notion of arbitrated prices that served to detect buying and selling opportunities (see fig. 3). This was essentially an analogue "app." Readers would input the price of one of two stocks they wanted to compare, which they were advised to take from the quotations "in the papers." Injecting this price in the table helped locate a line that gave the price at which other stocks would return the same yield. Comparing the quoted price for these

[129] In the words of Baily, "when we consider the great and extensive business which is constantly transacting in the purchase and sale of Annuities of various kinds [. . .] the subject assuredly acquires a degree of importance which it never before aspired to" (*Doctrine of Interest*, v–vi).

[130] On the mechanism for debt issues, see Cope, "Goldsmids." On the cost of Britain's war loans, see Newmarch, *Loans Raised by Mr. Pitt*. Newmarch's study had been completed with the assistance of Globe employee F. Hendriks, "whose skills and soundness as an actuary requires no praise of mine" (29–30), as Newmarch put it.

[131] Mortimer, *Every Man His Own Broker*.

FIGURE 3 ◆ MORTIMER'S TABLE OF EQUATION

TABLE of EQUATION,

Exhibiting the comparative Value *per Cent.* of the several Public Funds, in Proportion to the different Rates of Interest they bear—and to the current Price at the Stock Exchange, on any given Day, or Year.

Bank consols 3	S. Sea stock. 3½	Bank consols 4	Bank consols 5	Bank stock. 7	India stock. 10½	Annual Interest. £ s d
3pr cents at 48, are equal	to, at 56	64	80	112	168	6 : 5 : 0
49½	57¾	66	82½	115½	173¼	6 : 1 : 2
51	59½	68	85	119	178½	5 : 17 : 7
52½	61¼	70	87½	122½	183¼	5 : 14 : 3
54	63	72	90	126	189	5 : 11 : 1
55½	64¾	74	92½	129½	194¼	5 : 8 : 1
57	66½	76	95	133	199½	5 : 5 : 3

Source: Mortimer, *Every Man His Own Broker,* 1807, p. 26.

other stocks enabled to determine the cheapest asset. The Table of Equation automated the annuity theory.[132]

Against this backdrop, we discern signs of an emerging tension in the work of value adjudication. Until the late eighteenth century, Halley's formula had reigned supreme, but since it was approximate, it ran the risk of leaving money on the table. This was fine in a decentralized market, no longer so in a centralized one. Consider the following example: On December 30, 1796, the price of Long Annuities which matured in 1860 was quoted in the stock exchange's price list at £15.75, corresponding in round numbers to a yield of 6.2%.[133] Halley's direct formula gave a return of 6.9%.[134] Simpson's

[132] The granularity of Mortimer's table was coarse, and it did away with maturity. Mortimer (*Every Man His Own Broker*, unnumbered page toward end of preface) directs his reader to two alternative products: Blewert's *New and Easy Principle* and Fairman's *Stocks Examined*. Blewert's tables were initially published in 1783 with at least one subsequent edition, in 1804. Fairman's tables, initially published in 1795, had subsequent editions in 1796, 1798, 1802.

[133] In fact, between 15.65 and 15.875. See *Lloyd's List*, December 30, 1796.

[134] Of course, the horizon of the Long Annuity (which in December 1796 had a maturity of 63 years) extends past the threshold (40 years) beyond which Halley said his direct formula no longer produced precise results. To strictly follow Halley's recommendation, one would have to use here his algorithm's update function. However, popular guides, such as Dodson's *Anti-Logarithmic Canon* (53), Robertson's "Letter to James West" (514), and Callet's *Tables Portatives*

gave 6%. Whoever purchased this stock to sell it again at a profit had to bear in mind whether prospective buyers would be "Halleyans" or "Simpsonians."

The problem was rendered palpable by the fact that alternative English government securities that represented similar risks and should have returned the same yield, exhibited persistently different interest rates. One well-known phenomenon was the systematic overpricing of perpetual annuities (or Consols) compared to Long Annuities. Returning to the previous example, on December 30, 1796, 3% Consols traded at £55. Because these were, as said, perpetuities, calculation of the yield was trivial—it was the ratio of the coupon to the price, or 5.45%, significantly below the yield implied by the price at which Long Annuities traded, according to the metric provided by the conventional theory. This was regardless of whether the yield on Long Annuities was calculated "exactly" (6.2%), or with Halley's formula (6.9%), or with Simpson's formula (6%). Such pricing anomalies had of course always existed, but they emerged now in the open, disfiguring the price list.[135]

Direct formulae, still holding onto the thread of Halley's prestige, came under attack. In 1804, Francis Maseres published in *Scriptores Logarithmici* a critical review of Halley's work. Delving into Halley's update function, he was expressing a level of frustration toward the obscurity of the exposition and perplexity at Halley's faulty use of the science of algorithms that would have been out of place fifty years earlier. Maseres then derived the update function Halley ought to have produced, in fact using Simpson's converging series method. As already said, this led him to rediscover Simpson's update function. This came to the attention of stockbroker Baily who had gotten interested in the question of leases economics in 1802. Maseres's attack on the existing canon thus became the starting point of Baily's own exertions.[136]

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(71) exclusively presented Halley's direct formula, which was put forward as the general solution to the interest rate problem. These authors neglected Halley's algorithm entirely. The virality of Halley's direct formula, coupled with the prestige of its originator, might thus have led investors to use it beyond the domain of application as defined by Halley himself.

[135] Price, *Observations*, 1:287; Baily, *Doctrine of Interest*, 109; Thatcher, *Treatise on Annuities*, xiii; Odlyzko, "Novel Market Inefficiencies."

[136] Baily, *Purchasing and Renewing*, 1802. In *Purchasing and Renewing*, 1807, unnumbered page in preface, Baily states that the main alteration from the first edition was "the insertion of some new formula, in the Appendix, for finding the rate of interest in Annuities; a subject which has employed the pens of some of our ablest mathematicians. The author was led to the discovery of them by a perusal of the fifth volume of the *Scriptores Logarithmici*, by Francis Maseres, Esq. FRS and Cursitor Baron of the Court of Exchequer and, though they do not altogether belong to

Baily would espouse Maseres's criticism of Halley. As he would complain in the *Doctrine*, "[Halley's] theorems have been transcribed and formed into rules by various authors, but without any improvement or extension, down to the present time."[137] A reexamination of the numerical order was undertaken.

Like Automatons

If one was to shed Halley's direct formula, then the question became: In what numerical rituals ought the market be initiated? Jobbers, including Baily, were directly interested in the liturgy. This was because their profit margins and the precision of calculus comingled. Profitability hinged on successful inventory management or equivalently, proper setting of buy and sell prices. They bought aggressively in a rising market to sell again at new higher prices. The risk was being caught wrong-footed. This casts Baily's precision decree as a "jobber's idea." Promotion of a shared understanding would enhance the predictability of price movements, facilitating the work of jobbers. "Cognitive" heterogeneity was the enemy, because it translated into price variations large enough to eat into the jobber's "turn" (the margin between the bid and the ask prices). To give an idea of the magnitudes involved, a difference of 6 basis points in the calculation of the yield on the Long Annuity translated into a 1% price variation.

The mechanization of price making was a logical outcome. In fact, a striking feature of the world of jobbers, which contemporary accounts underscore, is that it was an automated one. The anonymous author of the *Art of Stockjobbing* (1816) described stockjobbers standing in the market "like machines, or automatons, to be *turned* into any shape the public think proper, either to buy or sell."[138] Descriptions of stockjobber David Ricardo liken him to a self-absorbed computing engine, a machine-like algorithm endowed with extrahuman ability to perform cross-security arbitrage: "He is said to have possessed extraordinary quickness in perceiving in the turns of the market any accidental difference, *which might arise between the relative price of different stocks, and to have availed himself of this advantage, so as to*

the subject of the present treatise, they are inserted for the use of those who attend to this branch of analysis."

137 Baily, *Doctrine of Interest*, vii.

138 Practical Jobber, *Art of Stock-Jobbing*, 23. Italics in original, preparing a pun in the next sentence (at the stock exchange the "turn" was the bid-ask spread or difference between a jobber's buying and selling prices).

realize as much as £200 or £300 in one day, by selling out of one and buying into the other or vice versa."[139]

Stockbrokers emerging as carriers of the numerical work of the capital market was a development that had been in the making for many years. In 1801, the organization of the London Stock Exchange into a common law voluntary association governed by a committee (which Baily would join) created an opportunity for market operators to become the stewards of value. For ages, ordinary stockbrokers had carried to the market the orders of their clients who, as a result, routinely turned to them for financial advice, which naturally included requests to weigh in on matters of pricing.[140] As a result, the government of the market, which jobbers had secured for themselves in 1801 by spearheading the transformation of the London Stock Exchange in a voluntary society, gave jobbers authority over brokers. Control of brokers facilitated the promotion of homogenous understandings of how to perform the work of value, not in a forceful way of course, since this would have been incompatible with the logic of voluntary association, but in a more subtle, educative fashion.[141]

This backdrop invites a final series of thoughts on the role of financial capitalism in laying the ground for the English numerical project as it would develop in the nineteenth century under the leadership of individuals such as Charles Babbage. Building on Ada Lovelace's famous description, previous research has emphasized how certain technological innovations, such as the Jacquard machine's "punch card" system, influenced the thinking of the "father of computing" and creator of the Difference Engine—hailed as the

[139] Quoted in Sraffa, *David Ricardo*, 10:73. Italics ours. Suggestively, this description could apply to modern market making, which rests on algorithms that detect disequilibria and turn them into profits. For an account that is particularly relevant to the present discussion, see MacKenzie, "'Making,' 'Taking'"; MacKenzie, *Trading at the Speed of Light*.

[140] Banner describes the forward-looking character of eighteenth-century understandings of asset pricing (*Anglo-American Securities Regulation*, 49–96). He is drawing on the anonymous author of the *Dialogue Between a Gentleman and a Broker*, which has the "Broker" educating a "Gentleman" in the forward-looking character of asset pricing, "for 'tis remarkable, that the Expectation of any Scheme hath, for the generality, greater Influence on the Stocks, than its [past] Accomplishment" (18). See also Dickson, *Financial Revolution*, 101–2.

[141] Finally, knowledge standardization was required for yet another reason: the main task of the Stock Exchange Committee was the supervision of debt restructurings and debt discharges in the context of stockbroker bankruptcies. We suspect that one function of Baily's first published intervention in financial calculus, namely, *Purchasing and Renewing*, 1802, was to serve as an instrument to assist with the determination and allocation of debts born out of the adjudication of default in the stock exchange.

first programmable analogue computer.[142] We have never found a mention, by contrast, of the influence which the numerical work of the London Stock Exchange might have had on Babbage. And yet a distinct lineage exists. A complete discussion would exceed the boundaries of this article but some elements are worth pointing out. They help us grasp the importance of the stock exchange as computing site and source of inspiration for Babbage's computation project and, beyond this, for the expansion of English capitalism's little acknowledged cyberpunk universe.[143]

Here the important point is that the stock exchange offered a natural outlet for the sons of private banking families. Private bankers (whether country banks or London banks) being in the habit of employing short-term funds in the stock exchange in the shape of repurchase agreements, it was important for them to develop proper connections.[144] Richard Baily, the father of Francis Baily, was a partner of the Baily and Vincent Bank in Newbury.[145] Likewise, Charles Babbage's father Benjamin was a partner with Praed, Mackworth and Babbage. Benjamin's links with the stock exchange community must have been extensive, because in 1802, he received one of the coveted proprietors' shares of the newly formed stock exchange. These highly prized instruments had been privately placed among the closest friends of the market, ensuring control. Jobbers had been the main recipients—one beneficiary had been James Penfold, Baily's partner. That Babbage's father got one implies he was considered a safe hand. This speaks volume of the tight links that must have existed between the Babbage family and the stock market.[146]

Against this backdrop, it is not surprising that Charles Babbage's initial trajectory indeed orbits Baily. To be sure, Babbage obtained the private actuarial tables of Baily's own dedicated computer George Barrett, after the death of the latter in 1821.[147] His subsequent interventions in the field of life insurance, culminating with the publication in 1826 of a tour d'horizon of the field, reflected the same commitment to computing accuracy that had in-

142 Lovelace, "Notes by the Translator"; Schaffer, "Babbage's Intelligence"; Ashworth, "Memory, Efficiency, and Symbolic Analysis."

143 See Flandreau and Legentilhomme, "Cyberpunk Victoria."

144 See Pressnell, *Country Banking*, 433, discussing the case of Berwick and Co.

145 Horton-Smith, *Baily Family*, 50.

146 London Stock Exchange, Deed of Settlement, MS 29,751, London Stock Exchange Archive. London Stock Exchange Membership Applications, 1802–1924.

147 Babbage did take care of building the actuarial tables for an insurance company project led by Baily in 1824. See Hyman, *Charles Babbage*; Brown, "Barrett, George."

spired Baily.[148] To these exhibits, we add the chapter "On Currency" in Babbage's celebrated *Economy of Machinery and Manufactures*. Its description of the London Clearing House, not coincidentally the interface between London bankers and the London Stock Exchange, is a reminder that financial capitalism was a major source of demand for computing services.[149]

The Institute of Actuaries

The third computing order came in the mid-nineteenth century and is associated with the creation, in 1848, of the Institute of Actuaries. It emerged because the British capital market was fast expanding and with it, its computing needs, which also complexified. Not only were more and better computers necessary, but they had to perform increasingly sophisticated operations exceeding the tasks stockbrokers could be programmed and trusted to deliver. The result was the emergence of a forerunner of modern financial engineering and its corollary, the creation of a more focused government of computers. As we proceed to show, "actuaries," a profession primarily specialized in the insurance business, became the pillars of this new order. As we argue, too, this development cannot be separated from the fact that insurance companies acquired a dominant position on the buy-side of the market for fixed-income securities, conferring peculiar import to their methods and theories.

Professionalization

We have already argued that, by bringing into being the concept of present value, the early modern science of wealth enabled the growth of life insurance. In practice, during a good part of the seventeenth century, a considerable amount of figuring out was still required to put together a company. As in the case of algorithms that existed to cultivate an ideal of truth, which in turn backstopped business interventions, De Moivre's probabilities, combined with the present value theory, secured a certain amount of trust in life insurance as business. As should have been the case, the theory remained silent on critical operational dimensions, not least because originally, the em-

[148] Babbage, *Comparative View*. Indeed, Babbage sought Baily's direction as he navigated the business of insurance. See Wilson, "Babbage Among the Insurers," 138.

[149] Babbage, *Economy of Machinery*, chap. 14. This very popular chapter was reprinted in the abridged version of Babbage's *Economy of Machinery* that was released soon after. See Babbage, *On Currency*.

pirical basis on which computations of insurance premia (the life tables) was fragile. This was the reason for the original "empiricism" which Dickson first emphasized—not to be understood however as opposed to "theory driven."

The business was best left to practitioners who would bear the liability. This helps explain the repeated emphasis we find in contemporary commentary on the importance of the character of promoters. In the absence of automation, prudence, honesty, and pragmatism remained cardinal virtues. Logically, too, the mathematicians of wealth remained at a safe distance from the burgeoning insurance industry, but no farther than a striking distance. The life insurance mania of the 1760s and 1770s provided an opportunity for a group of devout students of Halley, De Moivre, and Simpson to take up the cudgel against unsound numerical practices. Concomitantly, they started taking an active role in the actual design of companies. The prototype was the Equitable Society, conventionally hailed as the first actuary-run company, by which is meant run by a professional. The reason is that its statutes contained the first reference to the term "actuary," to describe the duties of the company's secretary—in fact its executive manager.[150] The position was occupied by mathematician Richard Price, whose *Observations on Reversionary Payments* (1772) soon became an authoritative reference. In fact, Richard Price can be considered as the leader of this group of computers who sought to discipline life insurance.[151]

As he was well aware—this is demonstrated by the many references the *Doctrine* makes to Price's *Observations*—Francis Baily was following on the steps of this generation of financial engineers.[152] Baily started with the problem of pricing leases, publishing in 1802 a treatment of this ubiquitous question.[153] The *Doctrine* then appeared in 1808, offering its high-end solutions to the canonical problem. In 1810, Baily dived into life contingencies with his *Doctrine of Life Annuities*.[154] This was really a course in structured finance, covering in detail all the theory that was needed to design insurance

[150] Ogborn, "Professional Name," 235–36, 246, stating that "actuary" was originally used in the statutes of the Society for Equitable Assurances on Lives and Survivorships as "synonymous for secretary when applied to the specialized business of life insurance." See also Morgan, *Equitable Society*.

[151] Bellhouse, *Leases for Lives*, chap. 8.

[152] This is illustrated, for instance, by his extensive references to Richard Price.

[153] Baily, *Purchasing and Renewing*, 1802.

[154] Ashworth claims *Life Annuities* was the "second edition" of Baily's *Doctrine of Interest*. "This edition is substantially different in detail and greatly extended," Ashworth adds ("Calculating Eye," 417n40).

companies in a scientifically rigorous way. Just before, Baily had spun off one chapter from the book ("Account of the Several Life-Assurance Companies Established in London") and published it as a stand-alone pamphlet. The "Account" was a survey of existing insurance companies. As the full title indicated, it assessed their individual "merits and advantages." The commitment of a company to abide by mathematical best practice was taken as a hallmark not only of rigor, but of probity and competence.[155]

In the wake of Baily's publications, actuaries started showing what M.E. Ogborn describes as the "marks of professional life." Thanks to the *Doctrine*, actuaries were becoming even better identifiable. Their skill, which enjoyed public recognition, could be transmitted with the help of Baily's pedagogical volumes.[156] Insurance company promoters received the message. They started to feel the need to try and placate actuaries, to avoid giving grounds for criticism. In 1824, for instance, Charles Babbage was offered a generous retainer by the board of the Protector Life Assurance company to become its actuary. As it turned out, the board was not interested in Babbage's input as much as in flashing the name of a mathematician.[157] As Baily emphasized to Babbage when he advised the latter on Protector's offer, Babbage would probably do very well since, on top of the retainer, he'd derive a sizable side income from "private practice" (consulting activities).[158] In short, significant resources started flowing toward computers.

At the end of the day, this defined what an actuary was: A group of individuals professing to operate according to the formal rules laid out by the stockbroker Francis Baily. The fact that the latter's project rested on the numerical infrastructure of the London Stock Exchange played no small part in its success. The market hosted a community of some of the most proficient computers of the time who were, as already explained, in a position to influence brokers, who in turn had access to individual clients seeking general financial advice. What the higher-ups in the market deemed sound was thus vested with a halo. In other words, the London Stock Exchange did entrench Baily's mathematics as the foundation of a knowledge system. In fact, its seal of approval became even more valuable as insurance companies

155 Baily, *Account of the Several Life-Assurance Companies.*

156 Ogborn, "Professional Name," 241. See also Alborn, "Calculating Profession." On professions in general, see Abbott, *System of Professions.*

157 Wilson, "Babbage Among the Insurers," 138–39.

158 Wilson, "Babbage Among the Insurers," 138; Ogborn discusses Baily's consulting work for Sun Life ("Professional Name," 240). See also Flandreau and Legentilhomme, "Cyberpunk Victoria," 1099.

started relying increasingly on the joint-stock format and sought to float their shares in the stock market.

The result was the Alliance British and Foreign Life Assurance Company, a piece of structured finance art launched in 1824, the gold standard of insurance companies as far as the respect of the canon was concerned. The project may be said to have been sponsored by the London Stock Exchange. The actuary was stockbroker Benjamin Gompertz, a friend of Baily who was soon to design the famous statistical law of mortality that bears his name. It was sponsored by stockbroker Moses Montefiore, a prominent member of the committee of the London Stock Exchange and Gompertz's brother-in-law. The company was further sponsored by N. M. Rothschild as well as by other prominent financiers such as Francis Baring and Samuel Gurney.[159] Their combined prestige helped successful placement of the company's capital of £5,000,000, but with it, just as in the case of the landed elite investing the equations of Halley and De Moivre, the profile of Baily's *Doctrine* was further elevated.

Brokers v. Actuaries

But the honeymoon between actuaries and the London Stock Exchange was bound to end. We have already discussed the persistence of (scandalous) "pricing anomalies." These cut a line between traders (stockbrokers) and long term investors (insurance companies). Originally, the take of Baily and his followers was that mispricing was an infantile disorder of financial capitalism. According to Baily, "the great inequality in the values that are given in purchases [of annuities] shows that the Public has not thought very accurately." He knew this, he explained, because he had "had frequent opportunities of observing this from the various cases which have been laid before him for solution." In fact, ordinary computers were liable to commit "mistake and error." Against this backdrop, his task consisted in rendering "the subject as plain and as simple to the generality of readers [through] accuracy and perspicuity."[160]

Actuaries thought that education would resolve the situation and saw infiltration of the London Stock Exchange as the way to go. But as it turned out, education did not suffice, and the price anomalies persevered. Taking stock of the nagging persistence of price distortions between Long Annui-

[159] Schooling, *Alliance Insurance*.

[160] Baily, *Purchasing and Renewing*, 1802, unnumbered pages in preface.

ties and Consols, George Thatcher—a mathematician employed in 1822 as clerk of stock exchange firm Amory, Barclay, Edward Ellis & Charles Laurence—published in 1825 an essay that argued the problem was not with the faulty operations of computers, but a "defect in theory."[161] As Thatcher put it, this "failure" of the common theory consisted in the fact that conventional tables (built with the help of the said theory) failed to help investors "judging when stock or annuities have been dear or cheap, as compared with each other; and that they have not pointed out the frequent opportunities, which are well-known as having occurred of making profits by exchanging and re-exchanging the one for the other."[162]

Data at hand, he showed that the respective price of Long Annuities and Consols, fixed-income securities of "similar maturities," representing the same credit risk, did not provide to the investor the same return. In fact, an investor who bought because the Long Annuity looked cheap might indeed make a wise decision, but only if he intended to hold onto the security. The cheapness of the Long Annuity was a structural phenomenon, not a short-term one. He might be sorely disappointed if he attempted to resell, for he might liquidate at an even lower price. In clear, while the "common theory" informed buy-and-hold investors adequately, it was of limited value for jobbers and arbitragers. What was needed was recovering the formula that governed secretly the relation between Consols and Long Annuities, a knowledge which Thatcher described as "profitable" and "confined to a few practical men alone."[163] Bearing witness of the import of such questions, a long list of stockjobbers (and bankers) is found endorsing Thatcher's quest.[164]

For their part, actuaries were not willing to throw out the baby with the bathwater. There was the fact, well known to them, that price discrepancies might reflect individual features of instruments—such as liquidity—which markets priced correctly in fact. If these features could not yet be absorbed

[161] The expression "defect in theory" appears in the long title of Thatcher's *Treatise on Annuities*. As stated in the book's introduction, the investigation had originated in the desire to understand the "remarkable deviations of the current price of marketable annuities from their value as deducible from the common theory" (xiii). London Stock Exchange Membership Applications lists.

[162] Thatcher, *Treatise on Annuities*, xiii.

[163] Thatcher, *Treatise on Annuities*, xiii. The long title of Thatcher's spoke of *showing when Annuities are Dear and Cheap.*

[164] See Thatcher, front matter, for the list of subscribers where jobbers (and bankers) predominated.

in the common theory, the common theory could still serve as a benchmark to amend with a few rules of thumb.[165]

Second, the bottom line of insurance companies consisted in ensuring proper intertemporal asset-liability matching. Because the liabilities were long-term, so had to be assets: Companies were buy-and-hold investors. In principle, of course, aggressive investment tactics might be deployed. But since this created scope for human errors, the norm was to avoid rotating the portfolio. In this sense, the insurance companies were a mathematical money-machine which, once tuned up, was let run its course and accumulate cash undisturbed. When they observed the discrepancy between Long Annuities and Consols, actuaries did not see market failure—they saw excess returns.

Finally, considered as an industry, assurance companies had collective norm-setting power. The funds at their disposal, if placed at the service of the right equations, would resolve the conundrum. Far from conceding that their formulae failed, actuaries argued defiantly that they were experts in the resolution of market failures. An early statement of the case is provided by Charles Jellicoe, a founding member and later president of the Institute of Actuaries. Jellicoe expounded this logic through what was at heart an anticipation of marginal calculus reasoning. As Jellicoe argued, actuaries were not just computing experts but experts in determining the "marketable" character of new securities.[166] Nothing, he argued, belonged "more peculiarly [to the] province [of the actuary] than the detecting the causes which may lead to the failure of a security, *and the supplying the means which are the most directly adapted to prevent such failure*." As for the "means" he referred to, they consisted in what he described as the companies' "readiness at any time to assume the burden [of holding the security] at a stipulated price." Because the

[165] Baily states that taxation penalized Long Annuities (*Doctrine of Interest*, 105–106); Thatcher admits the existence of conversion risks (*Treatise on Annuities*, xv). Chavaz and Flandreau, "High and Dry," shows the effect of liquidity on the price of fixed-income securities; by contrast, Odlyzko, "Novel Market Inefficiencies," expresses skepticism that liquidity factors explain the misalignment between Long Annuities and Consols.

[166] Jellicoe observed, "But, be the actual practice in making purchases of this description what it may, my object is, to insist upon the propriety of regarding them as investments [. . .] securing to the holder of them, in any case, a given rate of interest so long as he retains them. [. . .] These I believe to be the proper attributes of what are commonly called 'marketable securities' for, without such qualification it will generally be found that securities are not ordinarily saleable in the market" ("Contrivances Required," 160).

companies had "pricing power" they could provide a price floor, backstop the market.[167]

Taking as case in point "reversionary interests"—a form of event-contingent security—he noted that their price depended on the marginal buyer. It would be lower if the marginal buyer was a unit holder but higher if he was a large holder (because the latter would diversify the risk away). The result was that "investments have two values, dependent upon the circumstances of the dealers in them [. . .]. The societies are enabled to command the prices offered for isolated purchases, and thus secure for themselves the difference between the values to which I have alluded."[168] Pricing glitches would thus either resolve themselves naturally or they would enrich insurance companies who, as marginal buyers, would also be price setters. What is more, the machine was not just profitable: It was honest. As the formulae mechanized price making, they ipso facto operated as a check on exploitation of the pricing power.

Thus, while "brokers" were showing growing signs of impatience with the common theory, "actuaries" were eager to pile on. The matter came to a head in the 1830s, as insurance companies were being launched at a rapid pace that further accelerated following the Companies Act of 1844.[169] As this occurred, reports of delinquencies emerged. Instances of fraudulent practices, famously satirized by Dickens, were exposed, casting suspicion on the business overall.[170] In the past, the leaders of the new profession—Baily, Babbage, or later the actuary Henry Porter—had repeatedly expressed frustration regarding the "ignorance" of self-proclaimed actuaries.[171] If the profession was to provide its elite troops to the rising army of structured finance, it had to make sure to create a boot camp. Yet, despite the existence of a corpus, actuaries in the first half of the nineteenth century were still pretty much self-taught, resulting in significant heterogeneity. This provided opportunity to act. After denounc-

[167] Jellicoe, "Contrivances Required," 160–61; italics added.

[168] Jellicoe, "Contrivances Required," 159.

[169] We identify 91 life offices created between 1825 and 1843, and 56 between 1844 and 1848. See Deuchar, "Progress of Life Assurance"; Black, "Chronological and Statistical Chart."

[170] Supple, *Royal Exchange Assurance*, 138; Dickens, *Martin Chuzzlewit*.

[171] Baily speaks of widespread "ignorance or neglect of the true mathematical principles upon which [assurance companies] ought to proceed" (*Doctrine of Life-Annuities*, vii–viii); Babbage observed, "The degree of knowledge possessed by persons so situated at the different institutions is exceedingly various, passing through all degrees, from the most superficial acquirements, derived merely from the routine of an office, up to the most profound knowledge of the subject" (*Comparative View*, xi–xii); and Porter noted, "if this were true in 1826, how much more so it must be at the present time it is painful to contemplate" ("Education of an Actuary," 109).

ing a looming tragedy of the numerical commons, the Institute of Actuaries was created in 1848. Unlike the London Stock Exchange, this new institution would have extensive powers to secure the formatting of computers.

Supercomputer

Like the London Stock Exchange, the Institute of Actuaries was organized as a voluntary association. However, its design had more to do with Victorian learned societies. It established a journal, and organized periodic gatherings of members to read and discuss each other's papers or debate matters relevant to the society's project. In one respect, the institute distinguished itself from most other learned societies. Although initially the formation of the original group occurred through self-selection, a formal examination was added almost immediately, in 1850. Going forward, the aspiring fellow of the institute had to successfully sit a test certifying mastery of financial mathematics (interest, probabilities) as well as an understanding of the broader questions which the profession confronted.

The institute's original bylaws leave little doubt regarding what its progenitors had in mind: The new organization was meant to discipline financial calculus. As the statutes read, the goal was "elevating the attainments and status, and promoting the general efficiency of all who engaged in occupations connected with the pursuits of an Actuary."[172] As Samuel Brown, a founding member, would put the matter later, uniting the members of a profession in one body would help "keep down the pretensions of ignorant or unqualified persons."[173] So the institute's ambition was to define what constituted legitimate and proper actuarial thinking and practices, that is, to set the boundaries distinguishing the professional calculator from the amateur (or hack). To put it differently, the founders of the institute aspired to establish a property right over the term "actuary." Those who claimed the title "actuary" without having the required mathematical skills would be kept out where they'd be prevented from tarnishing the reputation of the craft.

That this vetting process was consubstantial to the project itself was demonstrated by the fact that it was inscribed in the association's first con-

[172] Institute of Actuaries, *Constitution and Laws*, 15.

[173] "Institute of Actuaries," 1867, 392. It added that the examination system ensured that "none but those duly qualified, and who had obtained a certificate form, and admission by the council, should be entitled to practice the profession" (392).

stitution.[174] The goal of the creators of the institute was not to tamper with the inner workings of assurance companies, nor (as had been the case when the stock exchange had gotten involved) to certify their business model. Rather, the institute sought to *calibrate computing equipment* and the examination system was a means to this end. The promoters of the institute saw this control over human machines as vital. The report on the first examination did not mince its words: It spoke of the fact that "upon this test will mainly depend the future position and character of the profession."[175] In other words, the test contributed to create and protect a brand on whose vitality the success of collective action depended.

Evidence shows that careful consideration had been given from the onset to the content of the examination, its structure, and, above all, its degree of difficulty. Initially, the exams could not be made too complicated, for it would have discouraged potential members to sit them, which would have hampered the growth of the society. But the council of the Institute (the association's governing body) had been careful to not make the exams too easy either.[176] Rates of admissions, imperfect as they are as a measure of selectivity, speak of an increasingly trying test. In the early years of the institute, most candidates passed. Over time, no doubt mirroring progress in the standing of actuarial profession, the institute found itself able to tighten the screw. In the 1860s, the success rate had declined to 50% and it kept falling.[177]

The initial plan hashed out before the council in July 1849 envisioned a single examination.[178] This was discarded two years later for a gradual "grooming" of candidates over a period of three years. Each cohort had to pass three exams, corresponding to three successive levels of studies, under-

[174] "The Council, at their first meeting subsequent to the Annual General Meeting, shall appoint two Fellows, an Official Associate, and an Honorary Member, or in lieu of the last, a third Fellow, as Examiners whose duty it shall be to examine candidates for certificates of competency in accordance with a plan to be approved by the Council" (Institute of Actuaries, *Constitution and Laws*, 22).

[175] "Institute of Actuaries," 1851, 110.

[176] "Being voluntary, [the exams] must not be made too difficult, or many will be deterred from offering themselves; nor must they be too easy, or the character of the Institute will suffer" (Porter, "Education of an Actuary," 117).

[177] For the first two years, all candidates (11) passed. The average rate of admission for the first decade was 87% (1850 to 1859). It fell to less than 50% in the next (1860 to 1869); data collected by the authors in the *Journal of the Institute of Actuaries*.

[178] *Council Minute Book*, vol. 1, 1848 to 1855, 79–83, Archives of the Institute of Actuaries, London.

taken while candidates apprenticed under the authority of a senior actuary in assurance companies or private practices.[179] The first two levels focused on fundamental bodies of knowledge (probability, algebra, geometry) and the specific techniques of financial calculus they founded. The third level tested the candidates' capacity to apply actuarial knowledge to current affairs, including questions on the practical workings of insurance companies or legal reforms. The control and uniformization of technical skills thus doubled up with an ideological control.[180]

As a result, teaching emerged as a key mission of the institute. Initially, aspiring actuaries training for the examination were given a reading list that included Baily's *Doctrine of Interest*, still the bible of the profession. But in 1872 it was decided that a full-fledged course ought to be attended.[181] Lectures were entrusted to William Sutton, actuary of the London and Provincial Law Assurance Company, a rising star of the institute.[182] The lectures were initially published in the institute's journal, and in 1882, Sutton's *Text-book* was released, updating Baily's *Doctrine*.[183] The *Text-book* was soon completed by an exercise handbook compiled by institute members Thomas Ackland and George Hardy and containing material selected, they explained, "from all available sources in Actuarial literature, and largely from the examination papers of past years."[184]

Taken together the textbook, exercise handbook, and examinations enabled a new level of standardization. For instance, it was during this period that the peculiar mathematical notations still used by actuaries today were established. New actuaries were like so many machines on which the institute could now install the diverse programs which its leaders—the financial engineers who dominated the organization—developed. A war was waged on "objectionable" methods which, through centralized control of the cur-

[179] "Institute of Actuaries," 1852, 195. As usual, this change was motivated in reference "to the great object for which the Institute was organized, viz., the elevation of the status and character of the profession" (195).

[180] As Thomas Sprague argued, "There was a great deal in the [third year's examination] that could not be learnt out of books, and (. . .) it [is] proper that it should be so, for the professional knowledge required in an actuary could be learnt from no books. Therefore, since the Institute gave certificates that certain persons were competent to act as actuaries, they did quite right to set questions which could only be answered by persons having a practical knowledge" ("Institute of Actuaries," 1868, 338).

[181] "Institute of Actuaries," 1872, 146.

[182] On Sutton's career, see Caverly and Bankes, *Leading Insurance Men*, 188.

[183] Sutton, "Lecture I"; "Lecture II"; "Lecture III."

[184] Ackland and Hardy, *Graduated Exercises*, 6.

riculum, could be weeded out. Obsolescence struck practices lying outside the corpus. From want of cultivation, they were marked for extinction.[185]

This points to the importance of the institute's research and development function, embodied in its journal, as a crucial cog of the numerical "food chain." The institute helped focus efforts by projecting an understanding of the research frontier, while at the same time creating symbolic rewards for virtuous achievements that helped push the frontier back. Owing to rapid sanction by the institute's research arm, successful derivation of a new formula or computing technique helped project status which could be cashed through remunerative consulting work. The Victorian era's great leap forward to extreme precision bears witness to the success of the endeavor. New formulae, once vetted, soon found themselves absorbed in the "taught knowledge."

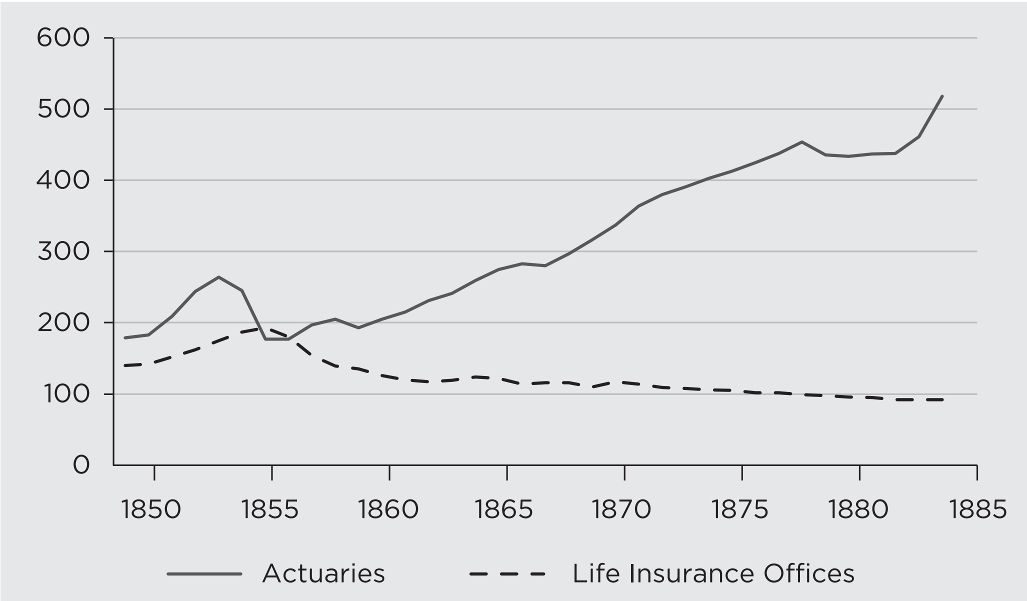
Formatting Computers, Powering Capitalism

Actuaries as a profession started providing the bulk of the financial computing services that came to be consumed during this cyberpunk phase of capitalist development. Having demonstrated stellar competency in sorting out the underlying assets, members of the institute were in position to become the new adjudicators of value. It is essential to emphasize, here, that the consequence of this was that the field of actuaries was not insurance but finance writ large: Bearing witness to this, actuary George King would entitle his 1882 handbook *Theory of Finance*. A further consequence was that the profession's reach exceeded its ostensible focus. Figure 4 shows that the stock of computers came to surpass by a significant margin the stock of insurance companies. As companies rarely needed more than one resident actuary (and in fact were sometimes happy to share him with another company, or employ him part time), the continuous secular increase (after the initial boom-bust) left a huge surplus of actuaries doing things other than "insurance stuff."

The financial tide of the 1850s and 1860s lifted the boats of the profession. Actuaries were dispatched to explore and colonize new territories. Market participants routinely sought actuarial input. The demand by classic consumers of actuarial advice such as the London Stock Exchange only expanded. So was the demand from banks, either because they needed to dress up loans for the market or because actuaries were found to be valu-

[185] *Miscellaneous Minute Book*, 1849 to 1873, vol. 1, fols. 102, 142, 152, 158, Archives of the Institute of Actuaries, London. See also Simmonds, *Institute of Actuaries*, 99.

FIGURE 4 ◆ NUMBER OF ACTUARIES AND OF LIFE INSURANCE COMPANIES, 1849–1884



Source: For the number of actuaries: annual reports of the Institute of Actuaries, and *Post Magazine Almanack*; for life insurance offices: Deuchar, *Progress of Life Assurance*.
Note: We include the fellows and associates of the Institute of Actuaries and its Scottish offshoot, the Faculty of Actuaries. Likewise, the number of offices also includes Scottish companies.

able managers. Actuaries were also used by building societies, accountancy firms, and they were involved, too, in the so-called finance companies which straddled the thin line between banking and money market funds. To give but one example, Arthur Scratchley, a prominent fellow of the institute, helped design the first "investment trust" and produced an influential theorization of the industry.[186]

Because of the police function computers were naturally led to play, the process had a self-reinforcing quality. Faced with financial innovations, the institute would react by developing formalisms that would elucidate the relation between the "canon" and the "derivative." A superalgorithm would be created that would address a whole class of products, of which the annuity

[186] See Flandreau and Legentilhomme, "Cyberpunk Victoria," esp. the section "Rise of the Computers."

would be a special case. The superalgorithm would thus light up the pathway for the computer, limiting misinterpretation and computing errors.

The descent of the celebrated Makeham's formula, propounded in 1874, stands as a characteristic example of this process of numerical colonization. It began in the mid-1860s with projectors attempting to originate a foreign government debt instrument that resembled the annuity. This novel animal differed from the standard in that reimbursement was by *equal installments*. Classic annuities by contrast involved constant payments. Broken down into its two components, there was an interest part and an amortization part, the first declining as more of the debt got reimbursed, and the second rising. By contrast, equal installments meant declining cash-outs. Since there was no known method to extract the yield in such cases, computers were sent back to brute force.

This invited a return of crude approximations leading to debate and confusion. In November 1865, a controversy erupted in the London *Times* regarding a new Austrian government loan that had this feature. The dispute started when a self-styled anonymous "Broker" wrote to the editor of the money article complaining that the loan was "usurious," a yield of 9.4%. He had arrived at this number by positing, rather than a gradual reimbursement, an average horizon at which the whole loan was repaid. This was Alexander's response to the Gordian knot.[187] A few days later, an "Actuary" responded deriding the Broker's mathematics and arriving as a result of an obscure reasoning at 7.5%.[188] It is likely that the "Actuary" was a front for the sell-side of the loan. The response from the institute was delivered by Peter Gray, who published subsequently in the *Journal of the Institute of Actuaries* an influential article that dealt with such loans. Gray was coming down on both Broker and Actuary, though more strongly on the latter, likely because he was off the mark, but also because he was abusing the label.[189]

Gray showed the "true" yield was 8.92018%. In fact, his paper provided the outline of a general method capable of determining, as Gray put it, "the exact [interest] rates, at the cost of no great expenditure of time or trouble" for the whole class of securities of which the Austrian loan was a specimen.[190] Gray's technique consisted in projecting the basic problem into an annuity

[187] "A Broker," *Times*, November 30, 1865.

[188] "An Actuary," *Times*, December 4, 1865.

[189] Gray, "Rate of Interest (Continued)," 187–88. Gray would nonetheless absolve the Broker, noting that despite his hardly orthodox method he had come "wonderfully near the truth."

[190] Gray, "Rate of Interest," 91.

formula that could be solved by parts. Equipped with Gray's equation, the actuary would have to derive, first, several intermediary parameters Gray called "the law of the annuity." These could then be plugged into a formula that enabled derivation of the yield.[191] There were many intermediary calculations involved and, what is more, their derivation mobilized numerical tools—such as difference operators—which students were barely familiar with.[192]

Gray's approach, tortuous as it was, was nonetheless a breakthrough. It showed that the bonds amortizable by equal installments could be colonized by the conventional numerical instruments. This was valuable for it enabled capitalizing on the computing knowledge accumulated thus far. This helps explain the enthusiasm shown by Gray, who spoke of his discovery as "the most general annuity theorem that has yet been given."[193] A few years later, mathematician William Makeham (another fellow of the institute) would complete the project.[194] Discussion of the detail of Makeham's formula and how it emerged would require a section of its own. Given the journal's space constraints, we'll just say that the formula's traction stemmed from its compact formalism, unambiguous identification of inputs and, especially, provision of clear commands minimizing human interpretation. To be sure, it imposed itself rapidly, its diffusion supported by the numerical enforcement powers of the institute.[195] It remains to this day a tenet of actuarial education.[196]

[191] The method relied on a formula Gray had discussed some years earlier in a note published in the institute's *Journal*. See Gray, "Tables," 191.

[192] "Report on the Class for the Second Year's Examination, 1872," in *Council Minute Book*, vol. 3, 1866 to 1877, 1–4, Archives of the Institute of Actuaries, London.

[193] Gray, "Tables," 191.

[194] See Jensen, "Makeham's Formula." Besides his formula, Makeham's name is associated with that of Benjamin Gompertz, whose equation of human mortality (according to which mortality risk grows exponentially with age) he enriched by inclusion of a constant term capturing the risk of death unrelated to aging. See Smith and Keyfitz, *Mathematical Demography*, 226.

[195] Sutton's textbook, published in 1882, dedicated a whole section to presenting the formula. Later textbooks drew almost entirely on Makeham's formalism when presenting the theory of redeemable loans: King, *Theory of Finance*, 1882, 60–66; Glen, *Actuarial Science*, 40–42; Todhunter, *Institute of Actuaries' Text-Book*, 72–76. As Todhunter's textbook was reedited three times between 1901 and 1937, Makeham's formula became de facto a mainstay in the diet of aspiring actuaries in the English-speaking world.

[196] See Garrett, *Introduction*, 134.

Coda: Rise of the Computer

This paper has made the case for financial mathematics as emic project of capitalism. The expanding use of financial mathematics harks back (at least) to the early modern age. It is best described not as a hoax by economists envious of mathematics but as a grassroots movement. It arose to assist trade in financial products and became the common language of exchange. Historians who will choose to continue to deride economics that-mathematizes-the-living will miss out on a fundamental insight which, as indicated, can be ascribed to Fernand Braudel: That the work of value is a considerable component of capitalist accumulation and that, accordingly, one ignores its significance at the peril of losing sight of something fundamental.

Whether lenders or borrowers, individuals caught by the "wheels of commerce" whirring with increasing loudness in the seventeenth and eighteenth centuries reacted by reaching out to mathematics and mathematicians. In a decentralized fashion, as a way to stabilize beliefs and adjudicate value, nascent capitalism put equations in the minds of traders. While there were of course many ways to grapple with the situation, the general tendency was a process of formatting. Here, the fundamental insight is that of the invisible hand (we don't mean it in the usual way) that designed the universe of finance (and thus the universe of ownership) in such a way that, despite an ostensible diversity, it remained calculable. In fact, under capitalism as it would develop from this point on, everything that existed would have to be computable.

Formatting of humans: People acquired the necessary formal skills, or more broadly learned to rely on tabulated algorithms inspired by the equations and which worked like apps. Or they sought guidance from those who understood the equations. Formatting of products: The universe of financial products is an ecology where survive instruments that can be sorted. Financial mathematics is not the science to study them. It is the merchant court in which they are adjudicated. Formatting of markets: Computing provides a market-making machinery. It is no surprise that the history of market making (the making of the London Stock Exchange, for instance) and the history of financial mathematics are joined by the hip.

Computers are the brokers of the capitalist system. Against this backdrop, the *longue durée* of financial computing is first and foremost, as we have seen, a story of the accumulation of computing power. Computing starts as adjudication, equity, and conscience. It starts from humble beginnings, almost invisible, informed by the idea of precision—the numerical equivalent of truthfulness. But absolute precision is never fully delivered, nor are algorithms really at work *inside* the wheels of financial commerce. They are a prayer and psalmody mumbled in their cabinet and sometimes proffered in their coffee house by the mathematicians of wealth. Algorithms as a prayer and psalmody, ultimately pointing to the religious origins of equilibrium.

From these beginnings, energized by the formidable power of the London Stock Exchange, the nineteenth century emerges with its rage for exactitude. Financial computing finds its lifeblood in the actuarial profession, endowed with tons of money and all the time there is. In the Victorian era, computing reaches a numerical summit, in the shape of superfast analogue convergence to an exact solution. We have showed the abaci that were necessary to enable reaching that moon, evoked the deep material transformations that took place in the City of London and how a new generation of computers was produced.

Circling back to our opening comment, the other change that occurred during the Victorian era was the inception of the neoclassical project. Given all that we said, it seems impossible to avoid emphasizing the striking resemblance between the most advanced prototype of computer extant—the Institute of Actuaries Fellow—and the calculating mind posited by nascent neoclassical economics, both subjects ready to make money from market inefficiencies. Suffice it to say at this stage that reference to physics envy is not necessary to account for the creation of an axiomatic of choice. *Homo economicus* seems to have been drawn after nature.

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Appendix ♦ Key tables covering the present value of £1 for any number of years

Author	Date	Maturity range (in years)	Maturity granularity (in years)	Interest rate range	Interest rate granularity (in basis points)	Annuity payable	Decimals
Simon Stevin	1582	1–30	1	1%–22%	100	Yearly	7
John Smart	1726	1–100	0.5	2%–10%	50 from 2% to 5%, 100 afterward	Yearly	8
Francis Corbaux	1825	1–100	1	3%–6%	25	Yearly, Half-Yearly, Quarterly	5
Peter Hardy	1839	1–100	1	0.25%–8%	25 from 0.25% to 5%, 100 from 5% to 8%	Yearly	4
Thomas George Rance	1852	1–100	1	0.25%–10%	25	Yearly	7
Lieutenant-Colonel W. H. Oakes	1877	1–100	1	0.75%–10%	125	Yearly	5

Source: See reference list.